

Review of set notation

$$A \cup B = \{x : x \in A \vee x \in B\}$$

$$A \cap B = \{x : x \in A \wedge x \in B\}$$

$$A - B = \{x : x \in A \wedge x \notin B\}$$

$$A \times B = \{(x, y) : x \in A \wedge y \in B\}$$

$$\bar{A} = A^c = U - A = \{x : x \notin A\}$$

$$P(A) = \{X : X \subseteq A\}$$

$A \subseteq B$ means for all $x \in A$,
 $x \in B$

OR
if $x \in A$ then $x \in B$

Today How to prove:

1. $a \in A$
2. $A \neq B$
3. $A \subseteq B$
4. $A = B$

How to prove $a \in A$:

In general, this involves showing a satisfies the definition of the set A .

Ex: if $a = \{2, 12, 17\}$, and $A = \{X \in P(\mathbb{N}) : |X| = 3\}$
prove that $a \in A$.

Proof: First, note that 2, 12 and 17 are in \mathbb{N} .

Thus $a \in \mathbb{N}$, and so $a \in P(\mathbb{N})$.

also $|a|=3$,

Thus $a \in A$. \square

Ex: if $C = \{3x^3 + 2 : x \in \mathbb{Z}\}$, then prove $-22 \in C$.

Proof: First, consider $x = -2 \in \mathbb{Z}$.

also $3(-2)^3 + 2 = -22$.

Thus $-22 \in C$. \square

$$x = 2$$

$$3 \cdot 2^3 + 2 = 26 \in C$$

$$x = 2 \in \mathbb{Z}$$

$$3 \cdot 2^3 + 2 = 26$$

$$x = -2 \in \mathbb{Z}$$

$$3 \cdot (-2)^3 + 2$$

$$-24 + 2 = -22 \in C$$

↑
this is scratch
work - does not
need to be included
in your proof

How to prove $A \not\subseteq B$

Give one example of $x \in A$ with $x \notin B$.

$$A = \{a, b, d, e\}$$

$$B = \{a, b, c, d\}$$

Is $A \subseteq B$? No -

because $e \in A$ but
 $e \notin B$.

How to prove $A \subseteq B$

Proof (direct): Suppose $x \in A$

:

Thus $x \in B$

□

Proof (Contrapositive): Suppose $x \notin B$

:

Thus $x \notin A$

□

Prove "if $x \in A$ then $x \in B$ "

conditional: $P \rightarrow Q$

contrapositive: $\sim Q \rightarrow \sim P$

contradiction: $\sim(P \rightarrow Q) = P \wedge \sim Q$

"Suppose $x \in A \wedge x \notin B$ "

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Ex Prove that $\{x \in \mathbb{Z} : 18|x\} \subseteq \{x \in \mathbb{Z} : 6|x\}$

Proof (direct) Suppose $a \in \{x \in \mathbb{Z} : 18|x\}$.

Then $a \in \mathbb{Z}$ and $18|a$, by the definition of the set.

then $a = 18y$ for some $y \in \mathbb{Z}$, by definition of divides.

$$a = 6 \cdot (3y)$$

Note $3y \in \mathbb{Z}$ by closure of \mathbb{Z} under multiplication.

thus $6|a$, by definition of divides.

Thus $a \in \mathbb{Z}$ and $6|a$.

Thus $a \in \{x \in \mathbb{Z} : 6|x\}$

□

Ex if A, B are sets, prove $P(A) \cup P(B) \subseteq P(A \cup B)$

Proof (Direct) Suppose $d \in P(A) \cup P(B)$

Then either $d \in P(A)$ or $d \in P(B)$

Case 1: Suppose $d \in P(A)$.

thus $d \subseteq A$

Then $d \subseteq A \cup B$

Thus $d \in P(A \cup B)$ \square

Case 2: Suppose $d \in P(B)$

thus $d \subseteq B$

and so $d \subseteq B \cup A$

so $d \in P(B \cup A)$ \square

How to prove $A = B$

Proof [1. prove $A \subseteq B$]

[2. prove $B \subseteq A$]

Thus, since $A \subseteq B$ and $B \subseteq A$, we have $A = B$. \square

Prop Given sets A, B and C ,

Prove $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$$A \times (B \cap C) \subseteq (A \times B) \cup (A \times C)$$

Proof (\subseteq , Direct) Suppose $(p, q) \in A \times (B \cap C)$

then $p \in A$ and $q \in B \cap C$, by definition of product.

also $q \in B$ and $q \in C$, by definition of intersection.

Since $p \in A$ and $q \in B$, we have

$$(p, q) \in A \times B$$

also since $p \in A$ and $q \in C$, we have

$$(p, q) \in A \times C.$$

so $(p, q) \in A \times B$ and $(p, q) \in A \times C$

Thus $(p, q) \in (A \times B) \cap (A \times C)$, by

definition of intersection

(3) Conversely, suppose

$$(p, q) \in (A \times B) \cap (A \times C)$$

:
:
:
:

Thus $(p, q) \in A \times (B \cap C)$