3.

#5

a. You have three vampire books, four steampunk novels, and five post-apocalyptic teen romances. How many ways are there of arranging them on a shelf if books of the same type must be kept together?

b. How many 5 element subsets are there from a set with 8 elements?

c. Find $|\{X \in P(A) : A = \{0, 1, 2, 3, 4, 5, 6\} \text{ and } |X| = 3\}|$

d. After expanding $(a + b)^5$, what is the coefficient of the term a^3b^2?

e. In a class consisting of 12 men and 15 women, how many different ways are there of choosing a group consisting of 3 men and 6 women?

f. What formula do we use to determine the number of subsets of size k of a set of size n? Write the formula, and describe what the different parts of the formula mean (where does the formula come from?).

number of k-element subsets of a set of size n:

n choose k, $_nC_k$, $\binom{n}{k}$

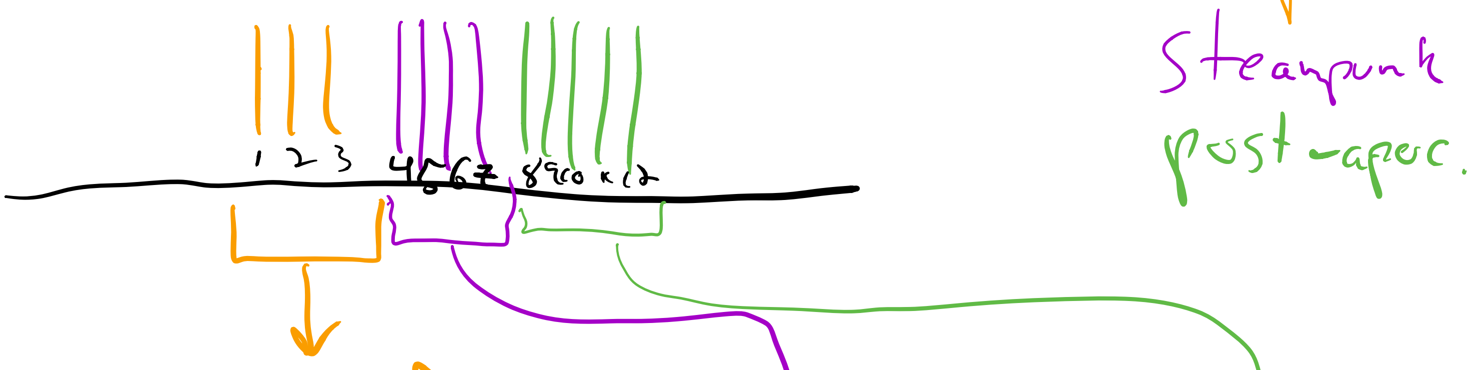
$$= \frac{n!}{(n-k)!\,k!}$$

3b) $n=8$, $k=5$

$$_8C_5 = \frac{8!}{(8-5)!\,5!} =$$

3c) $n=7$, $k=3$ $\quad {}_7C_3 = \frac{7!}{(7-3)!\,3!} =$

3d) $({}_{12}C_3)\cdot({}_{15}C_6)$



ways of arranging vampire books:

$\underline{3} \cdot \underline{2} \cdot \underline{1}$

$3! = \boxed{6}$

ways of arranging steampunk

$\underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$

$4! = \boxed{24}$

post-apoc.

$5! = \boxed{120}$

A	B	C
$\underline{3}$	$\underline{2}$	$\underline{1}$

$3! = \boxed{6}$ ways of arranging the genre's on the shelf.

total: $6 \cdot 6 \cdot 24 \cdot 120 =$ ______

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if P then Q

5. Proposition. For $x, y \in \mathbb{Z}$, if $x + y$ is even, then x and y have the same parity.

conditional two options:

Direct proof. Proof (Direct) Suppose P

⋮

Therefore Q

Contrapositive proof. Proof (contrapositive)

Suppose $\sim Q$

⋮

Therefore $\sim P$

Proof (Direct). Suppose $x, y \in \mathbb{Z}$ and suppose $x+y$ is even.

Thus $x+y = 2a$, $a \in \mathbb{Z}$, by definition of even

⋮

either x and y are both odd or x and y are both even

Therefore, x and y have the same parity.

Proof (contrapositive) Suppose $x, y \in \mathbb{Z}$ and x and y do not have the same parity.

Case 1 x is even and y is odd.

then $x = 2a$, $a \in \mathbb{Z}$ and $y = 2b+1$, $b \in \mathbb{Z}$ by definition of even, odd.

$x + y = 2a + 2b + 1$ by substitution

$= 2(a+b) + 1$

let $n = a+b$, then $n \in \mathbb{Z}$ since $\mathbb{Z}$ is closed under addition.

So

$x+y = 2n+1$, $n \in \mathbb{Z}$

Therefore $x+y$ is odd by defn of odd.

therefor $x+y$ is not even. $\boxed{\checkmark}$ QED

case 2: y is even and x is odd.

exactly the same proof but with x and y

switched,

→ instead of doing both cases, we say:

replace "case: x is even etc." with

Without Loss of Generality, x is even and y is odd.

1a) a and b are both positive.

<u>symbolize</u> how do we express "a is positive"

$P(a)$ = "a is positive"

$P(a) \wedge P(b)$

<u>negation</u>: $\sim P(a) \vee \sim P(b)$

$a > 0$ = "a is positive"

$(a > 0) \wedge (b > 0)$

neg:

$$\boxed{(a \ngtr 0) \vee (b \ngtr 0)}$$

or

$$\boxed{(a \leq 0) \vee (b \leq 0)}$$

New material 10/26

GCD, Euclid's Lemma.

Today: given $a, b \in \mathbb{Z}$

IDEA 1: all numbers that can be formed as linear combinations of a and b
(recall: a linear combo of a and b is an expression $ax+by$, $x, y \in \mathbb{Z}$)

IDEA 2: the Greatest common divisor of a and b, where $a, b \in \mathbb{Z}$ and at least one of them is nonzero, is the largest integer d such that

1. $d \mid a$ and $d \mid b$ ("d is a common divisor")
2. d is the largest common divisor, so if $e \mid a$ and $e \mid b$ ("e is a common divisor") then $d \geq e$.

Ex: consider $a=6$, $b=4$

what are some numbers we get as linear combinations of 6, 4?

$6x+4y$ $\quad x,y\in\mathbb{Z}$.

is 10 a linear combo of 6, 4?

Yes $10 = 6\cdot 1 + 4\cdot 1$

is 20 a linear combo?

$20 = 6\cdot 2 + 4\cdot 2$

$$\left.\begin{aligned} 40 &= 6\cdot 2 + 4\cdot 7 \\ 40 &= 6\cdot 6 + 4\cdot 1 \\ 40 &= 6\cdot 4 + 4\cdot 4 \end{aligned}\right\}\text{lots of ways}$$

what about 16?

$16 = 6\cdot 2 + 4\cdot 1$

$16 = 6\cdot 0 + 4\cdot 4$

$-6 = 6\cdot(-1) + 4\cdot 0$

Q what is the smallest positive

integer that we can obtain as a linear combo of 4, 6?

$$10 = 6\cdot 1 + 4\cdot 1$$
$$6 = 6\cdot 1 + 4\cdot 0$$
$$4 = 6\cdot 0 + 4\cdot 1$$
$$2 = 6\cdot 1 + 4(-1)$$
$$1 = ?$$

$$0 = 6\cdot 0 + 4\cdot 0$$

Theorem. Suppose $a, b \in \mathbb{Z}$ not both 0. If $d = \gcd(a,b)$, then

1. d is a linear combination of a and b
2. every linear combo of a and b is a multiple of d, and vice versa