Vocabulary

- theorem - proof - definition - proposition, lemma, corollary - even - odd	- parity - divides - divisor - multiple - direct proof

Definitions

- A **theorem** is a statement that is true, and has been proved to be true.
- A **proof** of a theorem is a written verification that a theorem is definitely and unequivocally true.
- A **definition** is an exact, unambiguous explanation of the meaning of a mathematical word, phrase, or symbol.
- *Words that mean the same thing as "theorem", but are used in special ways:*
    - *A statement that is true (and proven), but is not as significant as a theorem is sometimes called a* ***proposition***
    - *A* ***lemma*** *is a theorem whose main purpose is to help prove another theorem (a "little theorem, used along the way")*
    - *A* ***corollary*** *is a result that is an immediate consequence of a theorem or proposition ("a little something extra, that we get for free, having completed the theorem")*

Mathematical Definitions & Facts

- Definition. An integer n is **even** if $n = 2a$ for some integer $a \in \mathbb{Z}$.
- Definition. An integer n is **odd** if $n = 2a + 1$ for some integer $a \in \mathbb{Z}$.
- Definition. Two integers have the **same parity** if they are both even or both odd. Otherwise they have **opposite parity**.
- Definition. Suppose a and b are integers. We say that a **divides** b, written $a|b$, if $b = ac$ for some $c \in \mathbb{Z}$. In this case we also say that a is a **divisor** of b, and that b is a **multiple** of a.
- Definition. A natural number n is **prime** if it has exactly two distinct positive divisors, 1 and n.
- Definition. A natural number n is **composite** if it factors as $n = ab$ where $a, b > 1$.
- Fact. Suppose a and b are integers. Then so are $a+b$, $a-b$, and ab.

Proofs

A **theorem** is a statement that is true, and has been proved.

A **proof** is a written verification that a theorem is definitely and unequivocally true.

A **definition** is an exact, unambiguous explanation of the meaning of a word, phrase, notation.

$\boxed{\text{Proofs are about communication}}$

$\boxed{\text{It all comes down to definitions}}$

vocab: things that mean "theorem":

- proposition
- lemma
- corollary

Definitions

things already defined:

$\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$

$\in, \subseteq$

$+, \times, -, \div$

Defn: An integer n is <u>even</u> if $n = 2a$ for some integer a.

ex: is $n = 6$ even?

$\longrightarrow$ is 6 an integer? Yes (known)

$\longrightarrow$ is $6 = 2a$ for some integer a?

yes: $6 = 2 \cdot 3$, and 3 is an integer.

thus <u>6 is even</u>.

Is $x=0$ even?
0 is an integer,
$0 = 2 \cdot 0$
thus $x=0$ is even.

word/term: $\boxed{\text{even}}$ $\xleftrightarrow{\text{definition}}$ property: $\boxed{\text{its an integer and it equals } 2a \text{ for some integer } a.}$

Defn: An integer n is <u>odd</u> if $n = 2a+1$ for some integer a.

Theorem if a and b are integers, then so are:

$$a+b$$
$$a-b$$
$$ab$$

* $a \div b$ is not necessarily an integer.

Direct Proof

note: almost all theorems have the form "if P, then Q".

Proposition: $P \to Q$

Proof. Suppose P.

⋮

Therefore Q.

Direct Proof.

Ex:

Prop. If x is odd, then x^2 is odd.

Proof: Suppose x is odd

.

$\vdots$

Therefore, x^2 is odd