Exam Review
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Example 1

Translate to symbols. You may use the statements E(n) and O(n) in your answers.

    E(n): n is even
    O(n): n is odd

a. For every $n \in \mathbb{Z}$, $2n$ is even.
b. There is a subset X of $\mathbb{N}$ which has cardinality 5.
c. Every integer that is not odd is even.
d. For every real number x, there is a real number y for which $y^3 = x$.
e. Not all integers are even.
f. All integers are not even.

Example 3

a. Translate R(x) to symbols, using the statements P(x): x is prime, and S(x): x is a perfect square.

R(x): If x is prime then x is not a perfect square.

b. Translate S to symbols, using the set of even numbers E = {2,4,6,8,10,...}, and the set of prime numbers F = {2,3,5,7,11, …}.

S: Every even integer greater than 2 is the sum of two primes.

Example 4

Find the negation of the sentence, both in symbols and in words.

a. R: x and y are both odd.
b. S: All prime numbers are odd.
c. The square of every real number is non-negative.
d. For every real number x, there is a real number y for which $y^3 = x$.
e. If x is odd, then x^2 is even.

Vocabulary

- list - entry - length - empty list	- multiplication principle - repetitive and non-repetitive lists - factorial

Definitions and Notation

- A **list** is an ordered sequence of objects (called **entries** in the list). The **length** of a list is simply the number of entries. A list is typically written enclosed in parentheses, with objects separated by commas. Ex: (a,b,c,d,e) is a list of length 5.
  - *NOTE: order matters in a list, so* $(a,b,c,d,e) \neq (b,d,e,c,a)$
  - *NOTE: objects can be repeated in a list:* (a,a,b,c) *is a list of length 4.*
- The **empty list**, or list with no entries, is the only list with length 0.
- **Multiplication Principle**. Suppose in making a list of length *n* that there are a_1 possible choices for the first entry, a_2 possible choices for the second entry, a_3 possible choices for the third entry, and so on. The the number of different lists that can be made in this way is the product $a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_n$
- If *n* is a non-negative integer, then ***n* factorial,** written $n!$, is the number of non-repetitive lists of length *n* that can be made from *n* symbols.
- Theorem. The number of non-repetitive lists taken from a set of *n* symbols, with length *k*, is given by $\frac{n!}{(n-k)!}$

Example 1

Make a list of length 3 in which the first entry comes from the set $\{a,b,c\}$, the second entry comes from the set $\{3,4\}$, and the third entry comes from the set $\{a,x\}$.

Example 2

How many lists are there that satisfy the conditions of Example 1?

Example 3

Example 3: A standard license plate consists of three letters followed by four numbers. For example JRB-4412 and MMX-8901 are two different standard license places. How many different standard license plates are possible?

Example 4

Consider making lists from the set {A, B, C, D, E, F, G}. How many length-4 lists are possible if:
a) repetition is allowed? *"repetitive lists"*
b) repetition is NOT allowed? *"non-repetitive lists"*
c) repetition is NOT allowed and the list must contain an *E*?
d) repetition is allowed and the list must contain an E?

Example 5

Using the definition, calculate 3!, 2!, 1!, 0!. What is 4! ?

Example 6

This problem involves making lists of length 7 from the set {0, 1, 2, 3, 4, 5, 6, 7}.
a) How many such lists are there if repetition is allowed?
b) How many such lists are there if repetition is allowed *and* the first three entries must be odd?
c) How many such lists are there if repetition is not allowed?
d) How many such lists are there if repetition is not allowed *and* the first three entries must be odd?
e) How many such lists are there in which repetition **is** allowed, and the list **must** contain at least one repeated number?

Example 7

a) For which values of *n* does *n!* have *n* or fewer digits?
b) Using only pencil and paper, calculate $\frac{201!}{199!}$

Example 8

a) How many 6-digit positive integers are there in which there are no repeated digits, and all digits are odd?
b) How many 4-digit positive integers are there in which there are no repeated digits, and all digits are even?
NOTE: The number 0426 is NOT considered a 4-digit positive integer, since it is equal to 426.

Example 9

There are two 0's at the end of 10!=3,628,800. Determine the number of 0's at the end of 100!.

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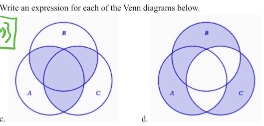
Exam 1 Review

Sec 1.1-1.8, 2.1-2.11 (except section 2.10 "negating statements", which will not be on the exam).
NOTE: On the exam you may be asked to write and explain your thinking (not just solve problems).

1. Write each of the sets by listing their elements between curly brackets.
   a. $\{x^2 - 2 : x \in \mathbb{N}\}$ b. $\{x \in \mathbb{Z} : |3x| < 15\}$ c. $\{7x - 1 : x \in \mathbb{Z}, |7x| \le 21\}$
2. Write each of the sets in set-builder notation.
   a. $\{..., -20, -15, -10, -5, 0, 5, 10, 15, 20, ...\}$
   b. $\{1, 4, 9, 16, 25, ...\}$
   c. $\{..., \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, ...\}$
3. Find the cardinality of each set.
   a. $|\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}|$ b. $|\{x \in \mathbb{Z} : |x^3| < 10\}|$
4. Label each statement True or False, and include a brief explanation.
   a. $\{\emptyset\} \subseteq \{1, 2, 3, \emptyset\}$ b. $\{\emptyset\} \in \{1, 2, 3, \emptyset\}$ c. $\{2, 4, 6, 8\} \in P(\mathbb{N})$
   For the remaining parts, let $D = \{7, \pi, \emptyset\}$ and $E = \{e, 3, 5\}$
   d. $\emptyset \in P(D)$ e. $(3, 7) \in D \times E$ f. $\{(7, 5), (\pi, \emptyset)\} \in P(D \times E)$ ← explain.
5. Given sets $A = \{a, b, c\}$, $B = \{c, d, e\}$, $C = \{c, e\}$ and universal set $U = \{a, b, c, d, e, f, g\}$, find each of the following and state the cardinality:
   a. $A \cup C$ b. $A \cap C$ c. $P(A)$ d. $(A \cup B) - C$
   e. $(A \cap B) \cup (B \cap C)$ f. $B \times C$ g. $\overline{A \cup B \cup C}$
   h. $P(C^2)$ i. $(B \times C) - C^2$
6. Given intervals $D = [1, 5]$, $E = (2, 6)$, and $F = [-3, 7]$,
   Write in interval notation: a. $D \cup E$ b. $F - (D \cup E)$ c. $\overline{D} \cap E$
   Sketch in the plane: $\boxed{\text{d. } D \times F}$ $\boxed{\text{e. } (F - E) \times \{1, 3, 5\}}$
7. Venn diagrams.
   a. Sketch a Venn diagram for: $A \cap (B \cup C)$
   *b. Sketch a Venn diagram for: $\overline{(A \cap B)} \cap C$

   Write an expression for each of the Venn diagrams below.



c. d.

8. Find the union and intersection of each collection.
   a. Let $A_1 = \{-2, 1\}$, $A_2 = \{-4, 2\}$, $A_3 = \{-6, 3\}$, and in general for each $n \in \mathbb{N}$, $A_n = \{-2n, n\}$. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$
   *b. For each $n \in \mathbb{N}$, let A_n be the closed interval of real numbers $A_n = [\frac{1}{n}, n]$.

$B \times C = \{(c,c), (c,e), (d,c), (d,e), (e,c), (e,e)\}$

$C^2 = \{(c,c), (c,e), (e,c), (e,e)\}$

$B \times C - C^2 = \boxed{\{(d,c), (d,e)\}}$

$|B \times C - C^2| = 2$

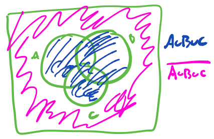
$A = \{a, b, c\}$

5c) $P(A) = \{\{a,b\}, \{a,c\}, \{b,c\}, \{a\}, \{b\}, \{c\}, \{a,b,c\}, \emptyset\}$

$|P(A)| = 8$

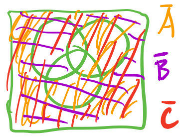
5g) $\overline{A \cup B \cup C} = \{f, g\}$

$|\overline{A \cup B \cup C}| = 2$



$\overline{A} \cup \overline{B} \cup \overline{C} = \{a, b, d, e, f, g\}$

$\{d, e, f, g\}$ $\{a, b, f, g\}$ $\{a, b, d, f, g\}$



$\frac{1}{2+3+5}$ vs. $\frac{1}{2} + \frac{1}{3} + \frac{1}{5}$

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   b. For each $n \in \mathbb{N}$, let A_n be the closed interval of real numbers $A_n = [\frac{1}{n}, n]$.
   Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$.
   *c. For each real number $\alpha \in [3, 6]$, let $A_\alpha = \{\alpha, 4\} \times [3, 4]$. Find $\bigcup_{\alpha \in [3,6]} A_\alpha$ and $\bigcap_{\alpha \in [3,6]} A_\alpha$ (give a description in words and a sketch).
9. For each item, determine if it is a statement. If so, determine whether it is True or False. If not, is it an open sentence?
   a. Frogs have yellow blood.
   b. $x^2 - 11 = 7$.
   c. Add 7 to both sides of the equation.
   d. $\mathbb{Z} \subseteq \mathbb{N}$
   e. Is $5 > 3$?
10. Write a truth table for each statement (show your work).
    a. $P \Rightarrow \sim (Q \wedge \sim P)$
    b. $(P \wedge Q) \Leftrightarrow (P \wedge R) \vee (Q \wedge R)$
11. Translate the given statement S into logical form, using statements P, Q and R.
    S: If $x \in \{4, 9, 12, 16\}$ and x is not a perfect square then $x = 12$.
    P: $x = 12$
    Q: x is a perfect square
    R: $x \in \{4, 9, 12, 16\}$
    $\boxed{(R \wedge \sim Q) \longrightarrow P}$
12. Determine whether each pair of statements is logically equivalent by comparing rows in a truth table.
    a. $P \Rightarrow Q$ and $\sim P \vee Q$
    b. $(P \wedge Q) \vee \sim R$ and $(R \Rightarrow P) \wedge (R \Rightarrow Q)$
13. Express (in English) the contrapositive of each statement.
    a. If a frog can jump, then it's alive.
    b. If x is a two-digit even number, then x isn't prime.

You may use the following in problems 14 and 15:
E(n): the number n is even
O(n): the number n is odd
$\mathbb{R}^+$: the set of positive real numbers
$\mathbb{R}^-$: the set of negative real numbers
F: The set of functions
G: The set of polynomial functions

14. Translate each statement to symbols. Is the statement true or false?
    a. For every integer x, the number $4x + 7$ is odd. $\boxed{\forall x \in \mathbb{Z}\ O(4x+7)}$
    "for every integer x" "for every x in the integers"
    b. There exists a real number x such that all real numbers greater than x are positive. $\exists x \in \mathbb{R}\ \forall y \in \mathbb{R}\ (y > x \longrightarrow y \in \mathbb{R}^+)$
    c. There is no real number y such that all real numbers less than y are negative.
    d. There exists a real number z such that all positive numbers are greater than z.
    e. For every function f, if f is a polynomial then f' is a polynomial. $\boxed{\forall f \in F (f \in G \longrightarrow f' \in G)}$
    f. If f and g are polynomials then so is the sum $f + g$.
15. Translate to English. Is the statement true or false?
    a. $\forall x \in \mathbb{R}, (x \in \mathbb{N} \Rightarrow x \in \mathbb{Z})$
    b. $\forall m, n \in \mathbb{Z} (E(m) \wedge O(n)) \Rightarrow O(m \cdot n)$
    c. $\forall f \in F, \sim (f \in G) \Rightarrow \sim (f = 2x^2 + 6x + 1)$

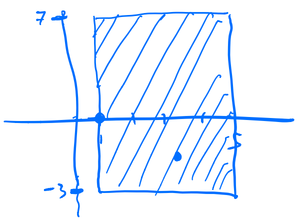
(i) $(B \times C) - C^2$
intervals $D = [1, 5]$, $E = (2, 6)$, and $F = [-3, 7]$,
interval notation: a. $D \cup E$ b. $F - (D \cup E)$
plane: d. $D \times F$ e. $(F - E) \times \{1, 3, 5\}$

$D = [1, 5]$

$F = [-3, 7]$

$D \times F = \{(x, y) : x \in D \text{ and } y \in F\}$

ex: $(1, 0) \in D \times F$
$(3.2, -1.1) \in D \times F$



*b. For each $n \in \mathbb{N}$, let A_n be the closed interval of real numbers $A_n = [\frac{1}{n}, n]$.

index set is $\mathbb{N}$,

$n = 2$, $A_2 = [\frac{1}{2}, 2]$ $|A_2| = \infty$

$n = 1$, $A_1 = [\frac{1}{1}, 1] = [1, 1]$ $A_1 = \{1\}$, $|A_1| = 1$

$n = 3$ $A_3 = [\frac{1}{3}, 3]$

$n = 10$ $A_{10} = [\frac{1}{10}, 10]$

$\bigcup_{n \in \mathbb{N}} A_n = \boxed{(0, \infty)}$

$\bigcap_{n \in \mathbb{N}} A_n = \boxed{\{1\}}$ $[1, 1]$ ok