Assignment2-Sec1.4-1.7: Problem 7

(5 points) Library/UMN/logicAndSetTheory/prob04.pg

This set is **visible to students.**

$|A| = \infty$

$A = \{-8, -7, \ldots, -1, 0, 1, \ldots$

Consider the universal set $\mathbb{R}$, define the interval $A = [-8, 1]$, interval $B = (-1, 5)$, and C be the negative real numbers. Complete the following exercises in interval notation.

$|B| = \infty$

$|C| = \infty$

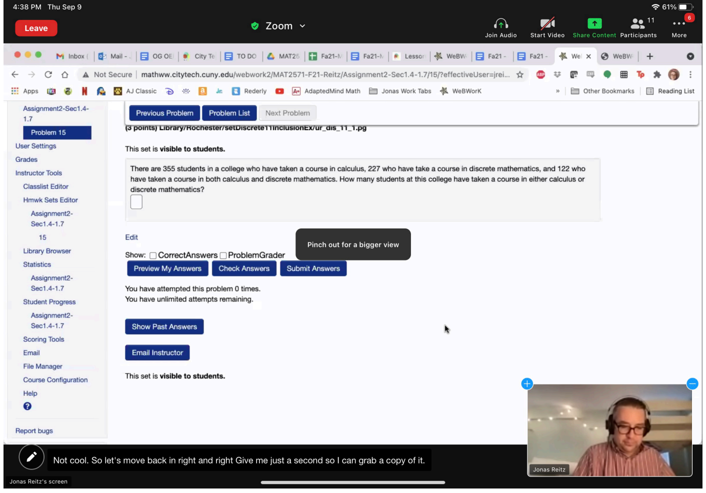
1. $A \cup B =$ ☐
2. $A \cap B \cap C =$ ☐
3. $A \cap (B \cup C) =$ ☐
4. $A^c \cup B^c \cup C^c =$ ☐
5. $(C - A) \cup (B - C) =$ ☐

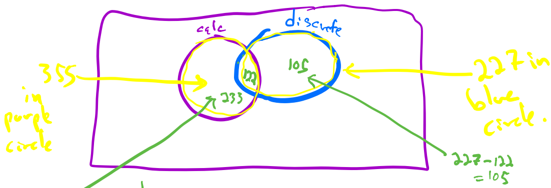
Note: *You may write the empty set* $\emptyset$ *as "EmptySet".*

$A \cap B \cap C = \{ -0.5,$
any # from -1 to 0
not including -1 or 0

$A \cap B \cap C = \boxed{(-1, 0)}$

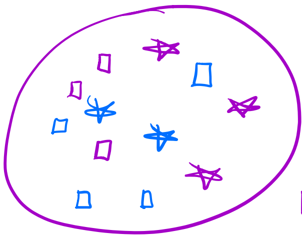
$A^c \cup B^c \cup C^c = \boxed{(-\infty, -1] \cup [0, \infty)}$





355−122=233 | 355+227 = 582

taken either = 233 + 122 + 105 = $\boxed{460}$

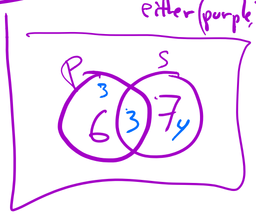


How many symbols altogether? 12

How many purple? 6

How many squares? 7

How many symbols are either (purple or square shaped)? 10



Vocabulary

- indices - indexed sets - index set *I*	- logic - correct logic vs correct information - statements

Indexed Sets

Definitions and Notation

- **Indexed sets** are sets that are distinguished by attaching subscript numbers instead of using different letters, such as A_1, A_2, A_3, A_4, A_5. We call the number 1,2,3,4 and 5 the **indices**.
- Unions and intersections of many sets. Suppose $A_1, A_2, A_3, \ldots, A_n$ are sets. Then
  - $A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n = \{x : x \in A_i \text{ for at least one set } A_i, \text{ for } 1 \le i \le n\}$
  - $A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n = \{x : x \in A_i \text{ for every set } A_i, \text{ for } 1 \le i \le n\}$
- Notation. Given sets $A_1, A_2, A_3, \ldots, A_n$, we define

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n \text{ and } \bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n$$

- Definition. The **index set *I*** is the set of all indices (of a collection of sets).
- Notation. If I is an index set, and for each $\alpha \in I$ we have a corresponding set A_α, then
  - $\bigcup_{\alpha \in I} A_\alpha = \{x : x \in A_\alpha \text{ for at least one set } A_\alpha \text{ with } \alpha \in I\}$
  - $\bigcap_{\alpha \in I} A_\alpha = \{x : x \in A_\alpha \text{ for every set } A_\alpha \text{ with } \alpha \in I\}$

Example 1

Suppose $A_1 = \{0,2,5\}$, $A_2 = \{1,2,5\}$ and $A_3 = \{2,5,7\}$. Find $\bigcup_{i=1}^{3} A_i$ and $\bigcap_{i=1}^{3} A_i$.

Example 2

Consider the following infinite list of sets:

$$A_1 = \{-1,0,1\},\ A_2 = \{-2,0,2\},\ A_3 = \{-3,0,3\},\ \ldots,\ A_i = \{-i,0,i\},\ \ldots$$

Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$.

Example 3

Let the index set I be the interval $[4,5)$, that is $I = [4,5)] = \{x : 4 \le x < 5\}$.

For each number $\alpha \in I$, let the set $A_\alpha = \{x \in \mathbb{R} : \alpha \le x \le 6\}$.

Find $\bigcup_{\alpha \in [4,5)} A_\alpha$ and $\bigcap_{\alpha \in [4,5)} A_\alpha$.

Example 4

Let the index set I be the closed interval $[0,2]$, that is $I = [0,2] = \{x : 0 \le x \le 2\}$.

For each number $\alpha \in I$, let the set $A_\alpha = \{(x,\alpha) : x \in \mathbb{R},\ 1 \le x \le 2\}$.

Find $\bigcup_{\alpha \in [0,2]} A_\alpha$ and $\bigcap_{\alpha \in [0,2]} A_\alpha$.

Sec 2.1 - Definitions (informal)

- The word **logic** refers to the way that humans *reason* **-** how we combine old information to deduce new information.
- A **statement** is a sentence or a mathematical expression that is either *definitely true* or *definitely false*.

Example 1

Suppose $A_1 = \{0,2,5\}$, $A_2 = \{1,2,5\}$ and $A_3 = \{2,5,7\}$. Find $\bigcup_{i=1}^{3} A_i$ and $\bigcap_{i=1}^{3} A_i$.

$$\bigcup_{i=1}^{3} A_i = A_1 \cup A_2 \cup A_3 = \boxed{\{0,1,2,5,7\}}$$

$$\bigcap_{i=1}^{3} A_i = A_1 \cap A_2 \cap A_3 = \boxed{\{2,5\}}$$

Example 2

$A_1 = \{-1, 0, 1\}$

$A_2 = \{-2, 0, 2\}$

$A_3 = \{-3, 0, 3\}$

$A_4 = \{-4, 0, 4\}$

$A_5 = \{-5, 0, 5\}$

$A_6 = \{-6, 0, 6\}$

$\vdots$

$A_{127} = \{-127, 0, 127\}$

$\vdots$

For each natural number i,

$A_i = \{-i, 0, i\}$

$$A_1 \cup A_2 \cup A_3 \cup A_4 \cup \ldots =$$

$$\{-1,0,1\} \cup \{-2,0,2\} \cup \{-3,0,3\} \cup \{-4,0,4\} \cup \ldots =$$

$$\bigcup_{i=1}^{\infty} A_i = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\} = \mathbb{Z}$$

$$\boxed{\bigcap_{i=1}^{\infty} A_i = \{0\}} \longrightarrow \text{or can write: } \bigcap_{i \in \mathbb{N}} A_i = \bigcap_{i=1}^{\infty} A_i = \{0\}$$

what is the set of all indices?

$I = \{1, 2, 3, 4, \ldots\} = \mathbb{N}$

Ex: consider $I = [2,4]$ the interval of the real line.

$|I| = \infty$

For $i \in [2,4]$,

let $A_i = \{(5,i)\}$

$A_3 = \{(5,\ )\}$

$A_{2.5} = \{(5,\ .5)\}$

$A_{2.55} = \{(5,2.55)\}$

$$\bigcup_{i\in I} A_i = A_2 \cup A_4 \cup \boxed{A_3} \cup A_{2.5} \cup A_{2.55} \cup \dots$$

$$= \{(5,3),(5,2.5),(5,2.55),\dots\} =$$

$$\bigcap_{i\in I} A_i = \emptyset$$