Vocabulary

- power set - union - intersection - difference	- universal set - complement - Venn diagram

Definitions and Theorems

- If A is a set, then the **power set** of A is another set, denoted $P(A)$, and defined to be the set of all subsets of A: $P(A) = \{X : X \subseteq A\}$.
- Theorem: If A is a finite set, then $|P(A)| = 2^{|A|}$.
- If A and B are sets, then
  - the **union** of A and B is the set $A \cup B = \{x : x \in A \text{ or } x \in B\}$.
  - the **intersection** of A and B is the set $A \cap B = \{x : x \in A \text{ and } x \in B\}$.
  - the **difference** of A and B is the set $A - B = \{x : x \in A \text{ and } x \notin B\}$ (sometimes written $A \setminus B$)
- If A is a set with universal set U, then the **complement** of A, denoted $\overline{A}$ (or A^C), is the set $\overline{A} = U - A$.

Example 1: If A={1,2,3}, find P(A).

Example 2

I. Is it a member of $P(\mathbb{N})$? a) {7} b) 7 c) {7,11,25,99} d) {-2,-1,0,1,2} e) *the even numbers* f) {2,3,5,7,11,13,...}	II. Is it a member of $P(\mathbb{R}^2)$? a) {(0,0), (0,1)} b) {0,1} d) The graph $y = x^2$, $\{(x,y) : y = x^2\}$ e) The graph of ANY function. f) *Sketch a silly picture* g) ANY black-and-white image in the plane

Example 3

Suppose $A = \{a,b,c,d,e\}$ and $B = \{d,e,f\}$

a) $A \cup B$ b) $A \cap B$ c) $A - B$

e) $(A - B) \cup (B - A)$ f) $(A \cap B) \cup (A - B)$ g) $(A \cap B) \times B$

Example 4

a) If P is the set of prime numbers, then what is $\overline{P}$?

b) If A={a, c, d, e, g} has universal set U={a, b, c, d, e, f, g}, then what is $\overline{A}$?

Example 5

I. For two sets A and B, sketch Venn diagrams for:	II. For three sets A, B, and C, sketch Venn diagrams for:
a) $A \cup B$	a) $A \cap B \cap C$
b) $A \cap B$	b) $(A \cup B) \cap C$
c) $A - B$	c) $A \cup (B \cap C)$

WHEN DO WE NEED PARENTHESES?

When an expression contains only unions $\cup$, then parentheses are optional.

When an expression contains only intersections $\cap$, then parentheses are optional.

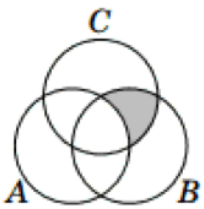
When an expression contains a combination of $\cup$ and $\cap$, parentheses are **essential**!

Example 6.

a) Sketch a Venn diagram for $(A - B) \cup (B \cap C)$

b) If $|D| = 10$, $|E| = 12$, and $|D \cap E| = 6$, what is $|D \cup E|$? *HINT: It might help to draw a Venn diagram for D and E, and figure out how many elements are in each region.*

c) Write an expression for this Venn diagram:



d) Consider the following intervals of the real line: $A = [1,4]$, $B = (2,5]$ and $C = (3,\infty)$.

i. Sketch the set $(A \cup B) \cap C$ on the number line.

ii. Sketch the set $(A - B) \times \overline{C}$ in the plane.

e) If $A = \{a,b\}$ and $B = \{a\}$, with universal set $U = \{a,b,c,d\}$, find:

i) $\overline{A} \times \overline{B}$ ii) $P(A) \cap P(B)$ iii) $P(B \times A)$ iv) $P\left(\overline{A}\right)$

Example 1: If A={1,2,3}, find P(A).

$\mathcal{P}(A) = \{\{1,2,3\}, \{1\}, \{\}, \{2,3\}, \{1,2\}, \{1,3\}, \{2\}, \{3\}\}$

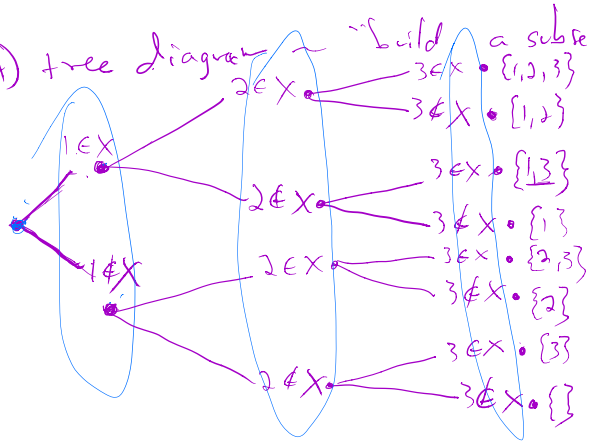
$|\mathcal{P}(A)| = 8$

$|A| = 3$

$\emptyset$

Theorem if $|A| = n$, $|\mathcal{P}(A)| = 2^n$

$\mathcal{P}(A)$ tree diagram — "build a subset $X \subseteq A$"



$1 \in A$

$1 \notin \mathcal{P}(A)$

$\{1\} \in \mathcal{P}(A)$ because $\{1\} \subseteq A$

$\{2,3\} \in A$?

$\{2\} \notin A$

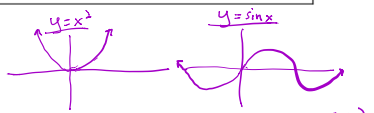
$\{2,3\} \subseteq A$? Yes

$\{2,3\} \in A$? NO

$\{2,3\} \in \mathcal{P}(A)$ Yes

Example 2

I. Is it a member of $P(\mathbb{N})$?	II. Is it a member of $P(\mathbb{R}^2)$? ($\mathbb{R}^2$ = the coordinate plane = $\{(x,y) \mid x \in \mathbb{R} \text{ and } y \in \mathbb{R}\}$; a set of ordered pairs of real #s.)
a) {7} Yes	a) {(0,0), (0,1)} Yes
b) 7 No — $7 \notin \mathcal{P}(\mathbb{N})$, 7 is not a set.	b) {0,1} No — not a set of ordered pairs!
c) {7,11,25,99} Yes	→ d) The graph $y = x^2$, $\{(x,y) : y = x^2\}$ Yes
d) {-2,-1,0,1,2} No — $-2 \notin \mathbb{N}$, $-1 \notin \mathbb{N}$, 0 ∉ ℕ	e) The graph of ANY function. YES
e) *the even numbers* No — depends on definition of even numbers	f) *Sketch a silly picture*
f) {2,3,5,7,11,13,...} Yes	g) ANY black-and-white image in the plane



Exam 1, Problem 1: ∈ $\mathcal{P}(\mathbb{R}^2)$

Example 3

Suppose $A = \{a,b,c,d,e\}$ and $B = \{d,e,f\}$

a) $A \cup B$ b) $A \cap B$ c) $A - B$

e) $(A - B) \cup (B - A)$ f) $(A \cap B) \cup (A - B)$ g) $(A \cap B) \times B$

a) $A \cup B = \{a,b,c,d,e,f\}$

b) $A \cap B = \{d,e\}$

c) $A - B = \{a,b,c\}$

d) $(A - B) \cup (B - A) = \{a,b,c\} \cup \{f\} = \{a,b,c,f\}$

$\{a,b,c\}$ $\{f\}$

g) $(A \cap B) \times B = \{d,e\} \times \{d,e,f\} = \{(d,e), (e,f), (d,f), (e,d), (d,d), (e,e)\}$

$(A \cap B) = \{d,e\}$

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prime number P: 3, 7, 13, ...

not a prime number $\overline{P}$: 4, 1, 21, ...

P = set of prime numbers = $\{3, 7, 13, \ldots\}$

P^c or $\overline{P}$ supposed to mean "everything not in P"

the universal set here is $\mathbb{N}$

complement = $P^c = \overline{P} = \{x : x \in \mathbb{N} \text{ and } x \notin P\} = \mathbb{N} - P$

Example 4

a) If P is the set of prime numbers, then what is $\overline{P}$?

b) If A={a, c, d, e, g} has universal set U={a, b, c, d, e, f, g}, then what is $\overline{A}$?

b) $\overline{A} = \{f, b\}$

Example 5

I. For two sets A and B, sketch Venn diagrams for:	II. For three sets A, B, and C, sketch Venn diagrams for:
a) $A \cup B$ b) $A \cap B$ c) $A - B$	a) $A \cap B \cap C$ b) $(A \cup B) \cap C$ c) $A \cup (B \cap C)$