

Day 1

Sec 1.1

Vocabulary

<ul style="list-style-type: none"> - set - element - cardinality 	<ul style="list-style-type: none"> - the sets \mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R} - the empty set - set builder notation
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Definitions

- A **set** is a collection of things. The things in the collection are called **elements** (or **members**) of the set.
- The notation " $x \in A$ " means "x is an element of A"
- Two sets are **equal** if they contain the same elements.
- The **cardinality**, or **size**, of a set A is the number of elements it has, and is written $|A|$.
- A set is called **finite** if it has a finite number of elements, otherwise it is **infinite**.

~~$|A| = \{1, 2, \dots\}$~~

Example 1

$|A| = 4$

$|B| = \text{infinity} = \infty$

$A = \{2, 4, 6, 8\}$ $B = \{1, 2, 3, 4, \dots\}$

$9 \in B$ but $9 \notin A$

- a. What does the "... " mean in set B?
- b. True or false: i. $2 \in A$ **T** ii. $5 \in A$ **F** iii. $5 \in B$ **T**
- c. True or False: i. $A=B$ **F** $A \neq B$ ii. $\{2, 4, 6, 8\} = \{8, 6, 4, 2\}$ **T** iii. $A = \{2, 2, 4, 6, 8\} = \{2, 4, 6, 8\}$ **T**
- Ignore duplicates
- d. What is $|A|$ (the cardinality of A)? What is $|B|$?

Example 2

Does $C=D$, where $C = \{\dots -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$, $D = \{0, 1, -1, 2, -2, 3, -3, 4, -4, \dots\}$?

Example 3

Sets don't have to consist of numbers.

$A = \{T, F\}$

$B = \{a, e, i, o, u\}$

$C = \{\text{students in this class}\}$

$D = \{(0,0), (0,1), (1,0), (1,1)\}$

$E = \{5, \{2,3\}, \{4,6\}\}$

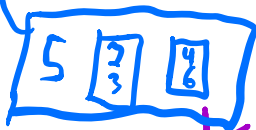
$|D| = 4$

$5 \in E$ **Y**

$\{2,3\} \in E$ **Y**

$2 \in F$ **NO**

2 is not an elt. of F



$|E| = 3$

Definition: The **empty set** is the set $\{\}$ with no elements. The symbol for the empty set is \emptyset .
NOTE: The symbol \emptyset is just shorthand for $\{\}$.

QUESTIONS:

- What is the size of the empty set $|\emptyset|$?
- Is $\{\emptyset\}$ the same as the empty set \emptyset ?
- True or False: i. Is $\emptyset \in \emptyset$? ii. Is $\emptyset \in \{\emptyset\}$?
- What is the size of the set $F = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}$?

$\{\} = \emptyset$
 $2 \in \{2, 3\}$
 $\{2, 3\} \in F$
 $|\{\emptyset\}| = 1$
 $|\emptyset| = 0$

Definition: In general, a set X written with **set-builder notation** has the form
 $X = \{\text{expression} : \text{rule}\}$
 where X is the set of all values of the expression that are specified by the rule.

Example 4
 List the members of each set. If the set is infinite, list a few members.

- $\{n^2 : n \in \mathbb{N}\} = \{1, 4, 9, 16, 25, 36, 49, \dots\}$
- $\{3n+1 : n \in \mathbb{Z}\}$
- $\{x \in \mathbb{Z} : 2x+7=3\}$ $n = 1, 2, 3, 4, 5, \dots$
- $\{2n : n \in \mathbb{Z}, |n| < 3\}$ $n = 1, 2, 3, 4, 5, \dots$ $d = \{2, 4, -2, -4, 0\}$
- $\{x \in \mathbb{R} : 1 < x \leq 5\}$ $x = 1, 2, -1, -2, 0$ not?

Write each set in set-builder notation:

- $\{2, 4, 8, 16, 32, 64, \dots\}$
- $\{\dots -15, -10, -5, 0, 5, 10, 15, \dots\}$
- the closed interval $[-3, 8]$ on the number line

Some Important Sets
 Some sets are important enough that they are given special names/symbols:

- The **empty set** $\emptyset = \{\}$
- The **natural numbers** are the set $\mathbb{N} = \{1, 2, 3, 4, \dots\}$
- The **integers** are the set $\mathbb{Z} = \{\dots -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$
- The **rational numbers** are the set of fractions $\mathbb{Q} = \{x : x = \frac{m}{n}, \text{ where } m, n \in \mathbb{Z} \text{ and } n \neq 0\}$
- The **real numbers** are the set \mathbb{R} (all the numbers on the number line)

\mathbb{N} \mathbb{Z}
 \mathbb{Q}
 \mathbb{R}