

Recall:  $f(x) = c \cdot b^x$ ,  $c, b \in \mathbb{R}$   
 $b > 0$ ,  $b \neq 1$

Example 15.1.

a)  $f(0) = 4, f(1) = 20$

c)  $f(2) = 160, f(7) = 5$

c)  $f(x)$  is exponential

$$f(x) = c \cdot b^x$$

$$f(2) = 160$$

plug in

$$160 = c \cdot b^2$$

$$f(7) = 5$$

plug in:

$$5 = c \cdot b^7$$

$$32 \frac{160}{5} = \frac{c b^2}{c b^7}$$

$$b^5 \cdot \underbrace{32}_b = \frac{1}{b^5} \cdot b^5$$

$$\frac{32b^5}{32} = \frac{1}{b^5}$$

$$b^5 = \frac{1}{32}$$

$$\sqrt[5]{b^5} = \sqrt[5]{\frac{1}{32}}$$

$$b = \frac{\sqrt[5]{1}}{\sqrt[5]{32}}$$

a)  $f(x)$  is exponential

$$f(x) = c \cdot b^x$$

$$\underline{f(0)=4}$$

↓  
plug in:

$$4 = c \cdot b^0$$

$$4 = c \cdot 1$$

$4 = c$

$$\underline{f(1)=20}$$

$$20 = c \cdot b^1$$

$$\underline{\frac{20}{4} = \frac{4 \cdot b}{4}}$$

$$\frac{20}{4} = 5$$

$5 = b$

$f(x) = 4 \cdot 5^x$

$$160 = c \cdot b^x$$

plugging in  $b = \frac{1}{2}$

$$160 = c \cdot \left(\frac{1}{2}\right)^x$$

$$4 \cdot 160 = c \cdot \frac{1}{4} \cdot 4$$

$$640 = c$$

$$\text{c) } f(x) = 640 \cdot \left(\frac{1}{2}\right)^x$$

**Example 15.3** The population size of a country was 12.7 million in the year 2000, and 14.3 million in the year 2010.

a) Assuming an exponential growth for the population size, find the formula for the population depending on the year  $t$  (where  $t = 0$  in the year 2000).

b) What will the population size be in the year 2015, assuming the formula holds until then?

c) When will the population reach 18 million?

year	$t$	pop
2000	0	12.7 million
2010	10	14.3 million

Population is exponential function

$$f(t) = c \cdot b^t$$

$$f(0) = 12.7 \text{ million}$$

$$12.7 = c \cdot b^0$$

$$\boxed{12.7 = c}$$

$$f(10) = 14.3 \text{ million}$$

$$\frac{14.3}{12.7} = \frac{(12.7) \cdot b^{10}}{12.7}$$

$$\therefore \sqrt[10]{\frac{14.3}{12.7}} = \sqrt[10]{b}$$

$$\sqrt[10]{\frac{14.3}{12.7}} = b$$

$$\left(\frac{14.3}{12.7}\right)^{10} = b$$

$$b \approx 1.012$$

$$\boxed{\text{population } f(t) = (12.7)(1.012)^t}$$

b) population in 2015?  $t = 15$

$$\text{substitute: } f(15) = (12.7)(1.012)^{15}$$

$$= 15.19 \text{ million}$$

people in  
2015

c)  $f(t) = 18 \text{ million?}$

$$18 = (12.7) (1.012)^t$$

find  $t$ .

$$\frac{18}{12.7} = (1.012)^t$$

recall:  $\log x = \log_{10} x$   
 $\ln x = \log_e x$

take  $\ln()$  of both sides:

$$\ln\left(\frac{18}{12.7}\right) = \ln(1.012)^t$$

$$\frac{\ln\left(\frac{18}{12.7}\right)}{\ln(1.012)} = \frac{t \cdot \ln(1.012)}{\ln(1.012)}$$

$$\frac{\ln\left(\frac{18}{12.7}\right)}{\ln(1.012)} = t$$

c)

$$29.24 = t$$

population reaches 18 million  
in 2029

## Exponential Functions - base e

$$f(x) = c \cdot b^x$$

$$e \approx 2.718$$

growth or decay

$$f(t) = P e^{rt}$$

$t$  = time (independent variable)

$P$  = initial amount

$r$  = growth rate ( $r > 0$ )  
decay rate ( $r < 0$ )

EXAMPLE: A study published about two weeks ago, on March 17th, in the New England Journal of Medicine found experimentally that the half-life of the Covid-19 virus in the air is approximately 1.15 hours. A single cough by an infected person can release up to 6 billion coronavirus molecules into the air. Let's consider what happens after a single cough by an infected person.

- a. Model the number of remaining virus molecules  $V(t)$  in the air at time  $t$  by an exponential function  $V(t) = Pe^{rt}$  (find  $P, r$ ).
- b. How many of virus molecules will remain viable 5 hours after the person coughed?
- c. How long will it take for the number of remaining molecules to reach 6 million (0.1% of the original amount)?

a)  $t=0, V = 6 \text{ billion}$

$$V(0) = 6$$

$$6 = Pe^{r \cdot 0}$$

$$6 = Pe^0$$

$$\boxed{6 = P}$$

$$V(t) = 6e^{rt}$$

half-life is 1.15 hrs.

after 1.15 hours,  $V$  is cut in half

$$t=0, V(0)=6$$

$$t=1.15 \quad V(1.15) = \frac{6}{2} = 3 \text{ billion}$$

plug in:

$$\frac{3}{6} = \frac{6e^{r \cdot 1.15}}{6}$$

$$\frac{1}{2} = e^{1.15r}$$

take  $\ln()$  of both sides

$$\ln \frac{1}{2} = \ln(e^{1.15r})$$

$$\frac{\ln \frac{1}{2}}{1.15 \cdot \ln(e)} = \frac{1.15r - \ln(e)}{1.15 \cdot \ln(e)}$$

$$\frac{\ln \frac{1}{2}}{1.15 \ln(e)} = r$$

$$\boxed{\ln e = 1}$$

$$\log_{10} = 1$$

$$\frac{\ln \frac{1}{2}}{1.15} = r$$

$$r \approx -0.603$$

$$V(t) = 6 e^{-0.603t}$$

$V$  = # of covid-19 molecules (in billions)  
 $t$ : time in hours

b) what is  $V(5) = ?$

$$V(5) \approx .29 \text{ billion molecules}$$

(290 million)

c) when will  $V$  reach 6 million?  
billion

when will  $V(t) = .006$  billion?

$$.006 = 6 e^{-0.603t}$$

solve for  $t$

$$\frac{.006}{6} = e^{-0.603t}$$

$$\ln\left(\frac{.006}{6}\right) = \ln e^{-0.603t}$$

$$\frac{\ln\left(\frac{.006}{6}\right)}{-0.603} = \frac{-0.603t}{-0.603} \quad \cancel{\ln}$$

$$\frac{\ln\left(\frac{.006}{6}\right)}{-0.603} = t$$

$$t = 11.46 \text{ hours}$$