4. Proposition. If $n \in \mathbb{N}$, then
$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + ... + n(n+1) = \frac{n(n+1)(n+2)}{3}$.
5. Proposition. $\sqrt{12}$ is irrational.
6. Proposition. Let $a, b \in \mathbb{Z}, n \in \mathbb{N}$. If $a \equiv b (mod\ n)$ then $a^3 \equiv b^3 (mod\ n)$.
7. Proposition. For any integer $n \geq 0$, it follows that $9 | (4^{3n} + 8)$.
8. Proposition. If A and B are sets, then $P(A - B) \subseteq P(A) - P(B)$.
9. Proposition. If m, n are integers, then $\gcd(m, n) \leq \gcd(m^3, n^3)$.

10. a. List the elements of the set: $\{x^2 : x \in \mathbb{Z}, |2x + 1| < 11\}$
    b. Find the cardinality of the set: $|\{\emptyset, 5, \pi, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, 1, \{\emptyset\}\}\}|$
    c. Given the set $D = \{0, 1, 2, \emptyset\}$, answer True or False (and explain):
    i. $\{\emptyset\} \in D$ ii. $\{\emptyset, 2\} \subseteq D$ iii. $\{(2, 0), (\emptyset, \emptyset)\} \subseteq D \times D$
    d. Draw a Venn diagram for $(A \cap B) \cup (A \cap C)$
11. Given sets $A = \{a, b, c\}$, $B = \{b, c, d\}$ and $C = \{a, b, e\}$, and universal set $U = \{a, b, c, d, e, f\}$, find each of the following sets and state the cardinality:
    a. $(A \cup B) - (B \cap C)$ b. $P(B - C)$ c. $B \times C$ d. $\overline{A \cap C}$
12. a. Let $A_1 = \{-1, 2\}, A_2 = \{-3, 4\}, A_3 = \{-5, 6\}$ and in general for each $n \in \mathbb{N}$, $A_n = \{-2n + 1, 2n\}$. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$.
    b. Let $\mathbb{R}^+$ be the set of positive real numbers. For each $\alpha \in \mathbb{R}^+$, let A_α be the closed

Proof. (Direct) Suppose $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$, and suppose $a \equiv b \bmod n$.

$n | b - a$ by defn of $\equiv \bmod n$

So $b - a = n \cdot p$, some $p \in \mathbb{Z}$ by defn of "$|$"
$+a \quad +a$

$b = np + a$

$b^3 = (np+a)^3$

$b^3 = (np+a)(np+a)(np+a)$

$b^3 = (n^2p^2 + 2npa + a^2)(np+a)$

$b^3 = n^3p^3 + n^2p^2a + 2n^2p^2a + 2npa^2 + npa^2 + a^3$
$-a^3 \quad -a^3$

$b^3 - a^3 = n^3p^3 + 3n^2p^2a + 3npa^2$

$b^3 - a^3 = n\underbrace{(n^2p^3 + 3np^2a + 3pa^2)}_{=q}$, $q \in \mathbb{Z}$ by closure of $\mathbb{Z}$ under $+, \cdot$

$b^3 - a^3 = n \cdot q$ some $q \in \mathbb{Z}$

$n | b^3 - a^3$

Thus $a^3 \equiv b^3 \bmod n$ $\square$

10. a. List the elements of the set: $\{x^2 : x \in \mathbb{Z}, |2x + 1| < 11\}$
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    d. Draw a Venn diagram for $(A \cap B) \cup (A \cap C)$

a) $x = \cancel{-6}, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, \cancel{5}$

$\{25, 16, 9, 4, 1, 0, 1, 4, 9, 16\}$

$\boxed{\{0, 1, 4, 9, 16, 25\}}$

$|2(-5) + 1|$
$= |-10 + 1|$
$= |-9|$
$= 9 < 11$?

$|2(-6) + 1|$
$|-12 + 1|$
$11 \cancel{<11}$