Day 14

Chapter 5

Vocabulary

- contrapositive proof	

Proposition 1. If a is an integer, then either a is even or a is odd, but not both.

Proposition 2. Suppose $x \in \mathbb{Z}$. If $x^2 - 6x + 5$ is even, then x is odd.

Outline for Contrapositive Proof

Proposition. If P, then Q.

Proof. Suppose $\sim Q$.

…

Therefore $\sim P$.

Proposition 3. Suppose $x, y \in \mathbb{Z}$. If 5 does not divide xy, then 5 does not divide x and 5 does not divide y.

Prop If a is an integer, then a is either odd, or even, but not both.

Proof. Suppose $a \in \mathbb{Z}$.

Apply the division algorithm to a and 2

dividend divisor

Notice that $a \in \mathbb{Z}$ (given) and $2 \in \mathbb{Z}$, and $2 > 0$, and so by the division algorithm, there exist unique $q, r \in \mathbb{Z}$ with $a = 2q + r$, and $0 \le r < 2$.

what can r be?

r must equal 0 or 1.

If $r=0$, then $a=2q+0$

$a=2q$

So a is even, by definition of even

If $r=1$, then $a=2q+1$

So a is odd, by definition of odd.

By uniqueness of r, the number a cannot be both odd and even. $\square$

Contrapositive Proof (indirect)

If $p \to q$ is a conditional statement, the contrapositive is

$$(\sim q) \to (\sim p)$$

$$p \to q \equiv (\sim q) \to (\sim p)$$

Outline Contrapositive Proof

Prop. $P \to Q$

Proof. (contrapositive) Suppose $\sim Q$

$\vdots$

Thus $\sim P$ $\square$

$P \to Q$

Proof (contrapositive). Suppose $x \in \mathbb{Z}$.

Suppose x is not odd.

Then x is even (proven in class).

So $x = 2n$, $n \in \mathbb{Z}$, by definition of even.

$$x^2 - 6x + 5 = (2n)^2 - 6(2n) + 5 \quad \text{by substitution}$$

$$= 4n^2 - 12n + \underset{4+1}{5}$$

$$= 4n^2 - 12n + 4 + 1$$

$$= 2(2n^2 - 6n + 2) + 1;$$

let $m = 2n^2 - 6n + 2$, then $m \in \mathbb{Z}$ by closure of $\mathbb{Z}$ under $+, -, \times$.

$$x^2 - 6x + 5 = 2m + 1$$

≥ 0 $x^2 - 6x + 5$ is odd, by defn. of odd.

Thus $x^2 - 6x + 5$ is not even, proven in class. $\square$

OR by division algorithm

Proposition: Suppose $x \in \mathbb{Z}$. if $\boxed{P \to Q}$ $x^2 - 6x + 5$ is even, then x is odd.

Proof (direct). Suppose $x \in \mathbb{Z}$, and $x^2 - 6x + 5$ is even.

So $x^2 - 6x + 5 = 2n$, $n \in \mathbb{Z}$ by defn of even.

$$-2n \quad -2n$$

$$x^2 - 6x + (5 - 2n) = 0$$

quad Form: $x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1(5 - 2n)}}{2 \cdot 1}$

$$x = \frac{6 \pm \sqrt{36 - 20 + 8n}}{2}$$

$$x = \frac{6 \pm \sqrt{16 + 8n}}{2}$$

$$x = \frac{6 \pm \sqrt{4}\sqrt{4 + 2n}}{2}$$

factor

$x^2 - 6x + 5$

$(x-1)(x-5) = 0$

$x = 1$, $x = 5$

$x = 1$

$1^2 - 6 \cdot 1 + 5$

$1 - 6 + 5 = 0$

$x = 5$

$5^2 - 6 \cdot 5 + 5 = 0$

$\sqrt{16 + 4n}$

$\sqrt{4(4 + n)}$

$\sqrt{4} \cdot \sqrt{4 + n}$

$$x = \frac{6 \pm 2\sqrt{4+2n}}{2}$$

$$x = 3 \pm \sqrt{4+2n}$$

??

Thus $x = 2m+1$

Thus x is odd.

Guiding Principle:

Its easier to go from simple to complicated

Proposition 3. Suppose $x, y \in \mathbb{Z}$. If 5 does not divide xy, then 5 does not divide x and 5 does not divide y.

$P \rightarrow Q$

P: $5 \nmid xy$ $\qquad$ Q: $5 \nmid x$ and $5 \nmid y$

try direct:

Proof (direct): Suppose $5 \nmid xy$

$\underbrace{xy \neq 5n}$ for any $n \in \mathbb{Z}$

not obvious how to work with an inequation.

Thus $5 \nmid x$ and $5 \nmid y$

Proof (contrapositive)

$\sim(5 \nmid x$ and $5 \nmid y)$

Suppose $5 \mid x$ or $5 \mid y$.

case 1: $5 \mid x$

$\rightsquigarrow 5 \mid xy$

case 2: $5 \mid y$

$\rightsquigarrow$

Thus $5 \mid xy$ $\square$