Negating Statements

$\sim P$ "not P" or "it is not the case that P"

- $\wedge$
- $\vee$
- $\rightarrow$
- $\forall x$
- $\exists x$

conjunction: $P \wedge Q$, $\sim(P \wedge Q) = \sim P \vee \sim Q$

disjunction: $P \vee Q$, $\sim(P \vee Q) = \sim P \wedge \sim Q$

conditional: $P \rightarrow Q$, $\sim(P \rightarrow Q) = P \wedge \sim Q$

universal quantifier $\forall x P(x)$, $\sim \forall x P(x) = \exists x \sim P(x)$

existential quantifier $\exists x P(x)$, $\sim \exists x P(x) = \forall x \sim P(x)$

Rules for negating statements (quantifiers and conditional statements)

1. $\sim(\forall x \in S,\ P(x)) = \exists x \in S,\ \sim P(x)$
2. $\sim(\exists x \in S,\ P(x)) = \forall x \in S,\ \sim P(x)$
3. $\sim(P \Rightarrow Q) = P \wedge \sim Q$

Example 4

Find the negation of the sentence, both in symbols and in words.

a. R: x and y are both odd.
b. S: All prime numbers are odd.
c. The square of every real number is non-negative.
d. For every real number x, there is a real number y for which $y^3 = x$.
e. If x is odd, then x^2 is even.

a) $\sim R$: x is not odd or y is not odd

b) $S: \forall x\,(P(x) \rightarrow O(x))$

$\sim S: \sim \forall x\,(P(x) \rightarrow O(x))$ $\xrightarrow{\text{step 1}}$ $\exists x \sim (P(x) \rightarrow O(x))$ $\xrightarrow{\text{step 2}}$ $\exists x\ P(x) \wedge \sim O(x)$

"there must be a prime number that is not odd"

$\exists x\ P(x) \wedge \sim O(x)$

c) $\forall x \in \mathbb{R},\ x^2 \geq 0$

$\sim(\forall x \in \mathbb{R},\ x^2 \geq 0)$

$\boxed{\exists x \in \mathbb{R}\quad x^2 \ngeq 0}$

"there is a real number whose square is negative" (or is not ≥ 0)

d) $\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\quad y^3 = x$

$\sim(\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\quad y^3 = x)$

step 1 $\exists x \in \mathbb{R} \sim (\exists y \in \mathbb{R}\quad y^3 = x)$

$\exists x \in \mathbb{R}\ \forall y \in \mathbb{R}\quad \sim(y^3 = x)$

$\boxed{\exists x \in \mathbb{R}\ \forall y \in \mathbb{R}\quad y^3 \neq x}$

"There is a real number x such that for all real numbers y, $y^3 \neq x$"

e) $O(x) \rightarrow E(x^2)$

$\sim(O(x) \rightarrow E(x^2))$

$O(x) \wedge \sim E(x^2)$

"x is odd and x^2 is not even"

Day 8

Sec 3.1, 3.2

Vocabulary

- list - entry - length - empty list	- multiplication principle - repetitive and non-repetitive lists - factorial

Definitions and Notation

- A **list** is an ordered sequence of objects (called **entries** in the list). The **length** of a list is simply the number of entries. A list is typically written enclosed in parentheses, with objects separated by commas. Ex: (a,b,c,d,e) is a list of length 5.
  - *NOTE: order matters in a list, so* $(a,b,c,d,e) \neq (b,d,e,c,a)$
  - *NOTE: objects can be repeated in a list:* (a,a,b,c) *is a list of length 4.*
- The **empty list**, or list with no entries, is the only list with length 0. → ( )
- **Multiplication Principle**. Suppose in making a list of length n that there are a_1 possible choices for the first entry, a_2 possible choices for the second entry, a_3 possible choices for the third entry, and so on. The the number of different lists that can be made in this way is the product $a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_n$
- If n is a non-negative integer, then ***n* factorial,** written $n!$, is the number of non-repetitive lists of length n that can be made from n symbols.
- Theorem. The number of non-repetitive lists taken from a set of n symbols, with length k, is given by $\frac{n!}{(n-k)!}$

Example 1

Make a list of length 3 in which the first entry comes from the set $\{a,b,c\}$, the second entry comes from the set $\{3,4\}$, and the third entry comes from the set $\{a,x\}$.

Example 2

How many lists are there that satisfy the conditions of Example 1?

Example 3

Example 3: A standard license plate consists of three letters followed by four numbers. For example JRB-4412 and MMX-8901 are two different standard license places. How many different standard license plates are possible?

Example 1

Make a list of length 3 in which the first entry comes from the set $\{a,b,c\}$, the second entry comes from the set $\{3,4\}$, and the third entry comes from the set $\{a,x\}$.

Example 2

How many lists are there that satisfy the conditions of Example 1? $\boxed{12}$

Ex:

(a,4,a)
(a,4,x)
(a,3,a)
(a,3,x)
(b,4,a)
(b,4,x)
(b,3,a)
(b,3,x)
(c,4,a)
(c,4,x)
(c,3,a)
(c,3,x)

(□ , □ , □)

3 options, 2 options, 2 options

total # is: $3 \cdot 2 \cdot 2 = \boxed{12}$

Example 3

Example 3: A standard license plate consists of three letters followed by four numbers. For example JRB-4412 and MMX-8901 are two different standard license places. How many different standard license plates are possible?

list length = 7

(J, R, B, 4, 4, 1, 2) = JRB-4412

□ □ □ □ □ □ □

26 26 26 10 10 10 10

total #: $26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

$= 26^3 \cdot 10^4 = \boxed{175{,}760{,}000}$

Example 4 → 7 items

Consider making lists from the set {A, B, C, D, E, F, G}. How many length-4 lists are possible if:

a) repetition is allowed? *"repetitive lists"*

b) repetition is NOT allowed? *"non-repetitive lists"*

c) repetition is NOT allowed and the list must contain an *E*?

d) repetition is allowed and the list must contain an E?

Example 5

Using the definition, calculate 3!, 2!, 1!, 0!. What is 4! ?

a) □ □ □ □ ex BCDC

$7 \cdot 7 \cdot 7 \cdot 7 = 7^4 = 2401$

b) □ □ □ □ ex: BCDA

ex: EDAB

$7\ 6\ 5\ 4 = 7\cdot6\cdot5\cdot4 = 840$

c) □ □ □ □ ex: EDCA, BAEC

where is the E?

① E □ □ □ $= 6\cdot5\cdot4 = 120$ ex: EABD (6 5 4)

② □ E □ □ $= 6\cdot5\cdot4 = 120$ AEDC (6 5 4)

③ □ □ E □ $=$ 120 DFEG (6 5 4)

④ □ □ □ E $=$ 120 CDAE (6 5 4)

$\boxed{480}$

d) ① E □ □ □ $= 7^3 = 343$ ex: EDCD, EDAF (7 7 7)

② □ E □ □ $= 7^3 = 343$ ex: AECB, AEAA

③ □ □ E □ $= 7^3 = 343$ ex: DBEB

④ □ □ □ E $= 7^3 = 343$ ex BBBE

1372 WRONG

ex EDAC → ①

DDEA → ③

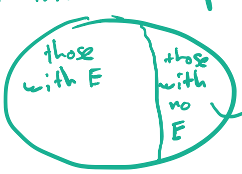
EDAE → ① and ④

we are counting this list twice!

Stuck – need new idea

d) repetition OK, must have E

All lists with repetition $= 7^4 = 2401$



list length 4 W/O E:

□ □ □ □ $= 6^4 = 1296$ (6 6 6 6)

$2401 - 1296 = \boxed{1105}$ ✱