Day 6

Sec 2.4, 2.5, 2.6

Vocabulary

- open sentence - converse - if and only if	- logically equivalent - contrapositive - De Morgan's Laws

Definitions and Notation

- A sentence whose truth depends on the value of one or more variables is called an **open sentence**. An **open sentence** is not a statement.
- The statement $Q \Rightarrow P$ is called the **converse** of $P \Rightarrow Q$.
  NOTE: A conditional statement and its converse express entirely different things!
- $P \Leftrightarrow Q$ means $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$. It is read "P if and only if Q".
- List of alternative phrases, all of which mean "$P \Leftrightarrow Q$"
  - P if and only if Q
  - P is a necessary and sufficient condition for Q.
  - For P is it necessary and sufficient that Q.
  - If P, then Q, and conversely.
- Two statements are **logically equivalent** if their truth values match up line-for-line in a truth table. In symbols, we express this using the equals sign.
- RULE: We are allowed to replace a statement with a logically equivalent statement
- The **contrapositive** of $P \Rightarrow Q$ is $(\sim Q) \Rightarrow (\sim P)$.

Example 1: Truth table for $P \Leftrightarrow Q$

P	Q	$P \Leftrightarrow Q$ $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$
T	T	
T	F	
F	T	
F	F	

Example 2

Is $P \Leftrightarrow Q$ logically equivalent to $(P \wedge Q) \vee (\sim P \wedge \sim Q)$**?**

Lesson 6: Biconditionals and Logical Equivalence

conditional "if... then"

$P \to Q$ conditional

$Q \to P$ the converse of

Question: do $P \to Q$ and $Q \to P$ mean the same thing?

NO

Truth table

$(P \to Q) \wedge (Q \to P)$

P	Q	$P \to Q$	$Q \to P$	$P \leftrightarrow Q$
T	T	T	T	T
T	F	~~F~~	~~T~~	F
F	T	~~T~~	~~F~~	F
F	F	T	T	T

Sometimes it's the case that both $P \to Q$ and $Q \to P$ hold.

P: n is even

Q: n is divisible by 2

$$\boxed{\underbrace{(P \to Q)}_{\text{cond.}} \underbrace{\wedge}_{\text{and}} \underbrace{(Q \to P)}_{\text{converse}}}$$

the biconditional

$$P \leftrightarrow Q$$

Find Truth table for biconditional:

Two statements are **logically equivalent** if their truth values match up line-for-line in a truth table. In symbols, we express this using the equals sign.

Ques: is $P \leftrightarrow Q$ logically equivant to "P and Q are both true, or P and Q are both false"

is $P \leftrightarrow Q$ equivalent to $(P \wedge Q) \vee (\sim P \wedge \sim Q)$? Yes, they are logically equivalent because the truth tables are identical

P	Q	$P \leftrightarrow Q$	$(P \wedge Q) \vee (\sim P \wedge \sim Q)$
T	T	T	T T F F F
T	F	F	F F F F T
F	T	F	F F T F F
F	F	T	F T T T T

Conclusion: $P \leftrightarrow Q = (P \wedge Q) \vee (\sim P \wedge \sim Q)$

Example 4: Which of the following is logically equivalent to $P \implies Q$?

1. $Q \implies P$ (the converse) NO
2. $\sim Q \implies P$
3. $Q \implies \sim P$
4. $\sim Q \implies \sim P$ (the contrapositive)

P	Q	$P \to Q$	~~$\sim Q \to P$~~	~~$Q \to \sim P$~~	$(\sim Q) \to (\sim P)$
T	T	T	F T	F F	T
T	F	F	T T	T F	F
F	T	T	F T	T T	T
F	F	T	T F	T T	T

logically equivalent

$$\underset{\text{conditional}}{P \to Q} = \underset{\text{contrapositive}}{(\sim Q) \to (\sim P)}$$

$$\sim(P \wedge Q) = \; ??$$

$$\sim(P \vee Q) = \; ??$$

De Morgan's Laws.