

Day 4

Sec 1.8, 2.1

Vocabulary

- indices - indexed sets - index set I	- logic - correct logic vs correct information - statements
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Definitions and Notation

- **Indexed sets** are sets that are distinguished by attaching subscript numbers instead of using different letters, such as A_1, A_2, A_3, A_4, A_5 . We call the number 1,2,3,4 and 5 the **indices**.
- Unions and intersections of many sets. Suppose $A_1, A_2, A_3, \dots, A_n$ are sets. Then
 - $A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n = \{x : x \in A_i \text{ for at least one set } A_i, \text{ for } 1 \leq i \leq n\}$
 - $A_1 \cap A_2 \cap A_3 \cap \dots \cap A_n = \{x : x \in A_i \text{ for every set } A_i, \text{ for } 1 \leq i \leq n\}$

- Notation. Given sets $A_1, A_2, A_3, \dots, A_n$, we define

$$\bigcup_{i=1}^n A_i = A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n \text{ and } \bigcap_{i=1}^n A_i = A_1 \cap A_2 \cap A_3 \cap \dots \cap A_n$$

- Definition. The **index set I** is the set of all indices (of a collection of sets).
- Notation. If I is an index set, and for each $\alpha \in I$ we have a corresponding set A_α , then

- $\bigcup_{\alpha \in I} A_\alpha = \{x : x \in A_\alpha \text{ for at least one set } A_\alpha \text{ with } \alpha \in I\}$
- $\bigcap_{\alpha \in I} A_\alpha = \{x : x \in A_\alpha \text{ for every set } A_\alpha \text{ with } \alpha \in I\}$

Example 1

Suppose $A_1 = \{0, 2, 5\}$, $A_2 = \{1, 2, 5\}$ and $A_3 = \{2, 5, 7\}$. Find $\bigcup_{i=1}^3 A_i$ and $\bigcap_{i=1}^3 A_i$.

3 sets: A, B, C

127 sets: $A_1, A_2, A_3, \dots, A_{127}$

↑
index

indices - plural

Example 1

Suppose $A_1 = \{0, 2, 5\}$, $A_2 = \{1, 2, 5\}$ and $A_3 = \{2, 5, 7\}$. Find $\bigcup_{i=1}^3 A_i$ and $\bigcap_{i=1}^3 A_i$.

$$\bigcup_{i=1}^3 A_i = A_1 \cup A_2 \cup A_3 = \{0, 2, 5, 1, 7\}$$

$$\bigcap_{i=1}^3 A_i = A_1 \cap A_2 \cap A_3 = \{2, 5\}$$

set of indices = $\{1, 2, 3\}$ $\stackrel{I}{\text{index set } I}$

What if we have infinitely many sets?

Example 2

Consider the following infinite list of sets:

$$A_1 = \{-1, 0, 1\}, A_2 = \{-2, 0, 2\}, A_3 = \{-3, 0, 3\}, \dots, A_i = \{-i, 0, i\}, \dots$$

Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$.

what is the index set? $I = \{1, 2, 3, 4, 5, \dots\} = \mathbb{N}$

$$A_1 = \{-1, 0, 1\}$$

$$A_2 = \{-2, 0, 2\}$$

$$A_3 = \{-3, 0, 3\}$$

$$A_4 = \{-4, 0, 4\}$$

$$A_5 = \{-5, 0, 5\}$$

$$\vdots$$

$$A_{20} = \{-20, 0, 20\}$$

$$\vdots$$

$$A_{117} = \{-117, 0, 117\}$$

$$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup A_3 \cup A_4 \cup \dots$$

$$= \{-1, 0, 1, -2, 2, -3, 3, -4, 4, \dots\}$$

$$= \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

$$= \mathbb{Z} \text{ the integers.}$$

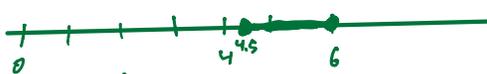
$$\bigcap_{i=1}^{\infty} A_i = \{0\}$$

Example 3
 Let the index set I be the interval $[4, 5)$, that is $I = [4, 5) = \{x : 4 \leq x < 5\}$.
 For each number $\alpha \in I$, let the set $A_\alpha = \{x \in \mathbb{R} : \alpha \leq x \leq 6\}$.
 Find $\bigcup_{\alpha \in [4, 5)} A_\alpha$ and $\bigcap_{\alpha \in [4, 5)} A_\alpha$.

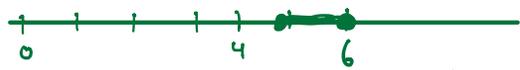
an example of an α :
 $\alpha = 4, A_4 = \{x \in \mathbb{R} : 4 \leq x \leq 6\}$.



$\alpha = 4.5, A_{4.5} = \{x \in \mathbb{R} : 4.5 \leq x \leq 6\}$

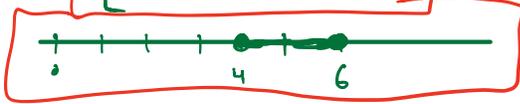


$\alpha = 4.99, A_{4.99} = \{x \in \mathbb{R} : 4.99 \leq x \leq 6\}$

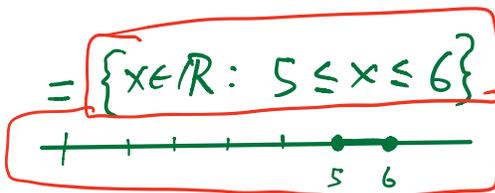


$$\bigcup_{\alpha \in [4,5)} A_\alpha = A_4 \cup A_{4.5} \cup A_{4.99} \cup A_{4.997} \cup A_{4.2683} \cup \dots$$

$$= \{4, 4.5, 4.99, \dots\}$$

$$= \{x \in \mathbb{R} : 4 \leq x \leq 6\}$$


$$\bigcap_{\alpha \in [4,5)} A_\alpha = A_4 \cap A_{4.5} \cap A_{4.99} \cap A_{4.237} \cap \dots$$

$$= \{x \in \mathbb{R} : 5 \leq x \leq 6\}$$


Example 4

Let the index set I be the closed interval $[0, 2]$, that is $I = [0, 2] = \{x : 0 \leq x \leq 2\}$.
 For each number $\alpha \in I$, let the set $A_\alpha = \{(x, \alpha) : x \in \mathbb{R}, 1 \leq x \leq 2\}$.

Find $\bigcup_{\alpha \in [0,2]} A_\alpha$ and $\bigcap_{\alpha \in [0,2]} A_\alpha$.

Sec 2.1 - Definitions (informal)

- The word **logic** refers to the way that humans *reason* - how we combine old information to deduce new information.
- A **statement** is a sentence or a mathematical expression that is either *definitely true* or *definitely false*.

Arithmetic
 Objects:
 numbers

Operations:
 $+$, $-$, \times , \div

Logic - study of human
reasoning

Objects: statements

Operations: combining statements
 to make more
 complicated statements