Day 2

Sec 1.2, 1,3

DO NOW: 100 ants (zero-length points) walk on a meter stick (a line) at 1 cm/second. When two ants collide, they both reverse direction. If an ant reaches the end of the stick, it falls off. What arrangement of ants maximizes the time before all ants have fallen off? How long can they last?

Vocabulary

- ordered pair - Cartesian product - ordered triple	- Cartesian power - subset

Definitions and Theorems

- An **ordered pair** is a list (x,y) of two things, x and y, enclosed in parentheses and separated by a comma. *NOTE: unlike a **set**, the **order** of the elements is important: (2,4) is NOT the same as (4,2).*
- The **Cartesian product** of two sets A and B is another set, written $A \times B$, and defined as $A \times B = \{(a,b) : a \in A,\ b \in B\}$.
- Theorem. If A and B are finite sets, $|A \times B| = |A| \times |B|$.
- An **ordered triple** is a list (x,y,z).
- A **Cartesian power,** like $\mathbb{R}^2$, is simply shorthand for the product of a set with itself $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ (similar for higher powers: $\mathbb{N}^3 = \mathbb{N} \times \mathbb{N} \times \mathbb{N}$)

Example 1: If A={p,q,r} and B={w,x}, find $A \times B$.

Example 2: i) Describe the Cartesian product $\mathbb{R} \times \mathbb{R}$.
ii) If A is the closed interval [0,1] and B is the half-open interval [2,3), draw a sketch of $A \times B$.

Example 3: If $A = \{3,7\}$, $B = \{2,4\}$, and $C = \{5,9\}$, then:
i) is $(3,2,9) \in A \times B \times C$? ii) is $(3,5,2) \in A \times B \times C$?

Definition

If A and B are sets and every element of A is also an element of B, then we say **A is a subset of B** and we write $A \subseteq B$. If this is *NOT* the case then we say **A is not a subset of B**, and we write $A \not\subseteq B$.
NOTE: $A \not\subseteq B$ means there is at least one element of A that is not an element of B.

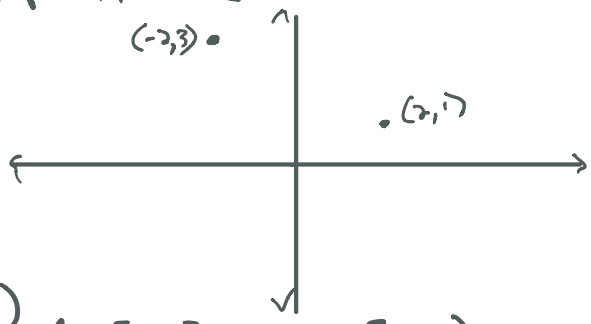
Example 1: If A={p,q,r} and B={w,x}, find $A \times B$.

$A \times B = \{(p,w), (p,x), (q,w), (q,x), (r,w), (r,x)\}$

$|A \times B| = 6$ $\quad |A| = 3,\ |B| = 2$

$|A \times B| = |A| \cdot |B|$

$\mathbb{R} \times \mathbb{R} = \{(a,b) : a \in \mathbb{R}, b \in \mathbb{R}\}$

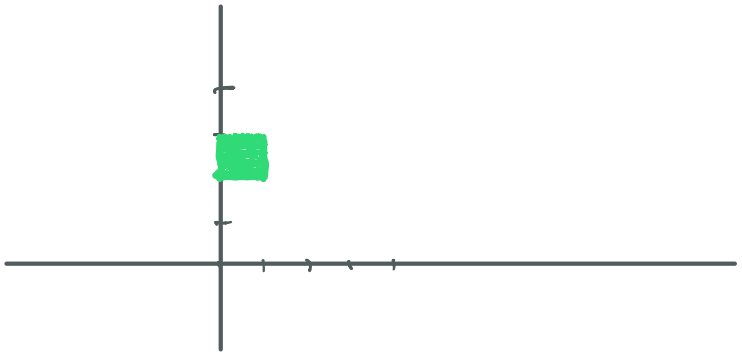


ii) $A = [0,1]$, $B = [2,3)$

$A \times B = [0,1] \times [2,3) = \{(a,b) : a \in [0,1], b \in [2,3)\}$

examples of members of $A \times B$:

$\{(0,2), (0.5,2.1), (0.68, 2.5), (0.00783, 2), (0.1, 2.999), \text{etc...}\}$



Example 3: If $A = \{3,7\}$, $B = \{2,4\}$, and $C = \{5,9\}$, then:
i) is $(3,2,9) \in A \times B \times C$? Yes ii) is $(3,5,2) \in A \times B \times C$? NO

$3 \in A$ ✓
$2 \in B$ ✓
$9 \in C$

$3 \in A$ ✓
$5 \notin B$ ✗

Intro to Proofs: Subsets

Definition
If A and B are sets and every element of A is also an element of B, then we say **A is a subset of B** and we write $A \subseteq B$. If this is *NOT* the case then we say **A is not a subset of B**, and we write $A \not\subseteq B$.
NOTE: $A \not\subseteq B$ *means there is at least one element of A that is not an element of B.*

Example 4

If $A = \{2,3,5\}$, $B = \{2,3,4,5,6,7,8\}$ and $C = \{1, 2, 3\}$

i) is $A \subseteq B$? Why? Yes - because every element of A is also an element of B.

ii) is $A \subseteq C$? Why? No - because $5 \in A$ but $5 \notin C$.

iii) is $C \subseteq A$? Why? No - because $1 \in C$ but $1 \notin A$.

iv) is $A \subseteq A$? Why? Yes - every element of A is also an element of A.

v) is $\varnothing \subseteq A$? Why? Yes - ~~b/c every element of $\varnothing$ is also an element of A.~~

OR we cannot find an element of $\varnothing$ that is not in A.

$\varnothing$ = empty set = $\{\ \}$