Math 2071 Introduction to Proofs and Logic, Section D638

Professor Reitz

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Group Process Paper

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The bridges and walking tours problem is a maze-like puzzle. The problem challenges the reader to try and create a plan of direction that will allow them to visit all the points or destinations in the problem, while passing over a bridge only one time. At a first glance, the problem looks like a children's maze or a puzzle that is of no sort of challenge for an adult. However, after a few minutes of working on the problem, the challenge is identified. After multiple attempts, and a few solutions, we each individually identified that there is no possible way to solve the problem.

The challenge of working to solve the original problem contributed to our first conversation discussing our conjectures. To begin, we all agreed about and shared the same conjecture relating to the original problem. Our united conjecture was that the original problem could not be solved and it proved to be true. The first group member's conjecture that we examined was Yasmine's, along with the puzzle that she made about her conjecture. Her puzzle was that of a cube with the vertices as points and the line segments as the bridges. After restlessly trying to solve her puzzle, she told us that it was not possible. I (Ahmad) am a bit stubborn when it comes to solving puzzles, so I made a few more attempts despite her saying that she built it to be impossible. When she said it was impossible to solve, it suddenly hit me that the puzzles we had to make did not have to be solvable. Before trying to solve her puzzle, I was under the assumption that everyone would make a solvable puzzle. This provided a lot of insight for me in regard to the puzzles and the overall concept of logic that is being discussed. Based on the puzzle that Yasmine made, and on the original problem that we were presented with, she came up with a conjecture. Her conjecture was that it is impossible to walk through all the bridges once if there are three or more bridges connected to one vertex. Based on our previous experiences with this kind of example, which were the original problem and Yasmine's own puzzle, it seemed like her conjecture was sound, accurate, and true. However, after Stephanie and I started to test the conjecture, we realized that the conjecture is false. In order to prove this, we came up with a very simple example of a puzzle that proves otherwise to Yasmine's conjecture.

Disproving Yasmine's Conjecture

Sticking to a puzzle designed in similarity to Yasmine's puzzle and her conjecture, we came up with a puzzle of three triangles. There is a point at each vertex, and each line segment between two vertices represents a bridge. In this puzzle, the three triangles are all connected at one vertex in the center. Therefore, this vertex has six bridges connecting to it. This puzzle has many different solutions. It can be solved using the center vertex with six connecting bridges as the starting point going towards any direction. It can also be solved using any of the other vertices as the starting point. While discussing Yasmine's conjecture, we concluded that an adjustment of the number of bridges connecting to one single vertex would not make the conjecture true. This is because any number of additional triangles and bridges can be added to the puzzle above and it would still be solvable in a variety of different ways. Yasmine was very surprised of this and also very surprised of the puzzle that we made in order to disprove the conjecture. Her surprise was very similar to mine when I was told that the puzzle she made was not solvable. We both had made assumptions regarding the concepts of the puzzle based on our own personal experiences with them.

The next puzzle and conjecture that we examined was Ahmad. Ahmad’s puzzle resembles that of an open envelope and is fairly easy to solve. His conjecture is related only to his puzzle and does not draw any inspiration from the original problem. When he was trying to solve his puzzle after making it, he noticed that although there are solutions to it, not any starting point will necessarily lead to a solution. So, my conjecture was, just because a puzzle has a solution, it does not mean that any starting point will result in a solution. An example that proves his conjecture to be true is my envelope puzzle. However, his conjecture is not absolutely true, it is conditionally true. There are some puzzles that can prove my conjecture to be true, like the envelope puzzle, however there are also puzzles that disprove my conjecture. A very simple triangle puzzle can be used to disprove his conjecture. In the triangle puzzle, there are a total of six possible solutions and three possible starting points. Any starting point that is selected will result in a total of two possible solutions. Therefore, his conjecture is conditionally true and does not apply to all puzzles.

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**C**

**E**

**D**

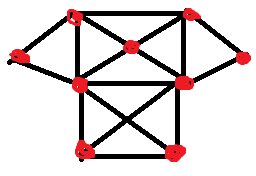
**B**

**A**

Ahmad's Envelope Puzzle Disproving Ahmad's Conjecture

|  |  |  |  |
| --- | --- | --- | --- |
| Lines | Vertices | How many with even degree | How many with odd degree |
| 8 | 5 | 3 | 2 |

Last but not least, we examined Stephanie's conjecture and puzzle. Stephanie's puzzle was by far the most complicated one. It included 17 Bridges and 9 points. Her puzzle was made up essentially of two squares and two triangles. Although her puzzle had a solution, it was too complicated for me to figure out due to the amount of bridges and points. After showing us the solution to her puzzle, I still wasn’t able to remember it and decided to swallow my pride and move on. Stephanie's conjecture, much like mine, was also related only to her own puzzle. Her conjecture stated, if we are able to start from any outside corner; then are we able to start from the only middle point. Her conjecture was also similar to mine in that we both mentioned the starting points of a puzzle. However, while trying to prove whether her conjecture was true or not, we came to the conclusion that it was not true. We used Stephanie's original puzzle to disprove her conjecture. By starting from the only middle point, we tried to solve the puzzle many times and at best, came up short by one bridge. Therefore, we concluded that out of the three conjectures, mine was the only one that was not absolutely false.



|  |  |  |  |
| --- | --- | --- | --- |
| Lines | Vertices | How many with even degree | How many with odd degree |
| 17 | 9 | 7 | 2 |

Stephanie's Puzzle / Disproving Stephanie's Conjecture

As a group, we decided that we would temporarily use Ahmad conjecture, and think about any adjustments or altercations that we can make to it. After meeting in class to re-discuss the conjecture, Stephanie and Yasmine (Perhaps Syed as well) enhanced our conjecture by asking a question instead of making a statement. The new conjecture reads, “What are the requirements for a point to work as a starting point?” While discussing the conjecture during our third meeting, we started to try and use our own puzzles that we previously made to see whether they can shed light to the question at hand. When observing Stephanie's puzzle, we already knew that the middle point cannot be the starting point because we had already tried to find a solution in that way and we were not able to do so. Therefore, we looked to some of the properties of the middle point and thought the answer could be lying somewhere in the middle of the square. The first thing that came to mind is the number of bridges that are connected to the point. In Stephanie's puzzle, the middle point has four bridges connecting to it. So, we mentioned that the number of bridges could be a factor in deciding the starting point of a puzzle. Yasmine and Stephanie took the analysis one step further and noticed that there was a significant difference between Stephanie's puzzle and the puzzle of triangles. That is, that each point that is connected to the middle point by one of four bridges, is also connected to at least three other points, not including the middle point. In the puzzle of triangles, the points stemming from the bridges connected to the middle point, are only connected to one other point / vertex.

**Our final Question: What are the requirements for a point to work as a starting point?**

As work to figure out what makes a starting point, our group went over some key definitions (special words) that helped us come up with a guess to prove this. First, we needed to know that a point is called a vertex, a line is called an edge, and the number of edges (lines) that lead to a vertex is called the degree. In other words, the number of lines connect to point is called the degree. Next, we used Ahmad’s puzzle to figure out why only two of his points works to be a starting point. We notice that his puzzle has 5 edges (lines) and 8 vertices, and point A has a degree of 2, point B and E has a degree of 4, and point C, D has a degree of 3. The two points that works as a starting point is point C and D. Next, we came up with a puzzle that has an even number of both vertex and edges, for example, 6 vertices and 10 edges.

-All points work as a starting point.

|  |  |  |  |
| --- | --- | --- | --- |
| Lines | Vertices | How many with even degree | How many with odd degree |
| 10 | 6 | 6 | 0 |

We saw that all starting points works if all points degree is even. As we try proving what makes a starting point, we come up with a guess or what we call our “rule.” **Rule one**, if the puzzle has even numbers of both vertices and edges, then all points must have an even degree, and therefore, all points work as a starting point. **Rule two**, if the puzzle has odd numbers of vertices and even numbers of edges, then two points must have an odd degree, this also goes for odd numbers of edges and even numbers of vertices. But for this rule, only two vertices can be odd degree, if there is more than two odd degree, then none of the points works as a starting point. Lastly, our **third rule** threw us off a bit because when we have two different numbers (odd numbers) of vertices and edges, we saw that two degrees of odd works as a starting point. But if we have the same number (odd number) of vertices and edges, we have that our degree is even. For example, we use three vertices and three edges (triangle), and all points gave us 2 degrees. This whole time, we thought that the number of vertices and number of edges matters to be a starting point. But in this case, it doesn’t matter if you have an even or odd number of vertices or edges. As a group, we came up with a guess for the conclusion, our conclusion to the question is that, to have a starting point you either need to have two vertices that the degree is odd or have no odd degree, which means all points have an even degree, therefore all points works as a starting point.