Bridge and Walking Tour

Conjecture: The minimum number of connecting lines in a Bridge Problem is equal to the total number of points minus one.

On the surface the Bridge Problem seems to be nothing more than a child’s game. It is possible to create Bridge Problem a child could solve, but it is also possible to create problems which would challenge the likes of Gary Kasparov and Deep Blue. Much like the classic Chinese game ‘Go’, every additional point, or intersections, that exists creates an exponential number of possible solutions. Also, like Go, Bridge Problems are not defined or limited to ant set number. Both games can have an almost infinite size of variables. This is true due to the near limitless space available in the virtual world. It was for this reason of near infinite possible solutions that I chose to address the minimum number of connecting lines. In other words, the simplest possible solution. To tackle this conjecture, the basic rules of the Bridge Game must be addressed. Because this problem can have varying rules it is necessary to discuss which rules we will be following and what rules, if any, we will be ignoring.

As we delve deeper into this problem we will need to nail down some terminology so that we can eliminate any confusion. For instance, the concept of points will be central to the conjecture. There exist three types of points. A starting, central, and ending point. No point may be more than one of these three classes. A starting point, (S), is the first point in a puzzle where the connecting line will begin, and is defined at a specific point. The ending point, (E), is the last point connected in a puzzle and is also defined at a specific point. The central points, (C), are neither starting nor ending points. The minimum number of central points is zero while there is no limit to the maximum number of central points. Each point of intersection can be connected to any point a maximum number of one times, there can be only one connecting line between any two points. All points must be visited and connected by one continuous non-repetitive line.  All points, except the ending point, can be visited more than one time, but when trying to follow the shortest path in a Bridge Problem we will want to use the minimum number of visits to each point. The starting point has a maximum number of connecting lines equal to the total number of points minus one. The minimum number of connecting lines for a starting point is equal to one. An ending point has a maximum and minimum number of connecting lines equal to one. A central point has the maximum number of connecting lines equal to the total number of points minus one total points minus two. The minimum number of connecting lines is equal to two, because central point must have at least two adjoining points to attach too. If there are less than three points, then no central points exists.

Because the starting point only has one line exiting it, in a solution looking for the shortest route(s), we will have only one connecting line attached to it, (S, C1). The ending point will also have only one line attached to it, when finding the shortest path, (C(last), E). While the central points will have only two connecting lines in this scenario, (C(previous), C(self), and (C(self), C(next). Therefore, for the starting and ending points there will be a total of one line attached to each. While all middle points (C) will have two connecting lines. Unfortunately, this format does not directly address how we handle lines that are not unique. For instance, which point do we attribute the line (C1, C2) to? This line, (C1, C2), is equivalent to a line going from point from C2 to C1, (C2, C1). This creates a situation where we could be counting a single line multiple times. The solution to this issue is to count each middle point as have one unique line total. Furthermore, for bookkeeping sake, we will consider all lines leading into a point as belonging to the previous point. Conversely all lines leaving a point will be attributed to the point which the line has just left. The ending point (E) is connected to a central point by one line which will have been already accounted as belonging to the last central point, (C(last), E), and thusly will have zero unique lines. Therefore, one line for (S) plus one line per (C) plus zero lines for (E), (S+C(total)+0E), will give us the minimum number of lines in a Bridge Problem, (S)+(C(total)) +(0E) = P(total)-1.

As I worked my way through this conjecture I realized the limitations any conjecture may have because of the rules or regulations associated with the material. This solution is no different. The formula I derived will only give us the minimum number of connecting lines to a problem, if we wanted to find the maximum number of solutions or some other number in between these two values then we would need independent equation. This formula also does not cover how many total solutions will have the minimum cardinality of connecting lines. This solution also becomes invalid if we lift the requirement that each point be limited to one of the three forms mentioned, starting, central, and ending. For instance, if a starting point is also an ending point then the equation is no longer valid. The new solution would be that the minimum number of lines would equal to the total number of points.

By systematically defining the rules and variables in The Bridge and Walking Tour puzzles I was able to derive a formula which, under specific conditions, will give the minimum number of connecting paths to any Bridge Problem. Other formulas, such as maximum number of connecting paths, could be derived from the information available. That, though, will have to wait as I have a bridge, or two, to traverse.