**Group Project – Mutilated Checker Board**

**Puzzle Description**

 Our group was introduced to a game known as Mutilated checker board. This game constitutes of dominos and a board. The board is similar to the used to play in a game of chess. A checkerboard is an 8x8 grid of squares (64 squares in total) colored alternating black and white and a domino is a 2x1 block that exactly covers the area of two squares on the checkerboard. The pictures of a checkerboard and a domino are shown in figure 1 and 2 below:

 

**Figure 1:** 8x 8 Checkerboard **Figures 2:** Domino

 Our goal of this game is to determine if the checkerboard can be covered perfectly with the dominos when any TWO squares are removed, regardless of the colors or locations. To complete this challenge there are rules to be followed stating the following:

1. Dominos cannot overlap each other and they must fit in the given space of the checkerboard.
2. Dominos can be aligned either vertically or horizontally but not diagonally. Therefore, as long as we do not violate the rules, any combination of the placements of the dominos is valid.

**Proof process narrative**

 The procedure of figuring out whether or not an incomplete board can be covered by dominos is a process of repetitive observations, thoughts being exchanged, and patient communications. We discussed different ideas, such as even and odd numbers, the relationship of being symmetric or not, the connection between the board and the dominos and the remaining areas of the board. Throughout this project, we as a group became able to determine, whether or not the dominos are able to cover the checkerboard using examples and various other ways leading to how we came up with a conjecture and a proof of it. Based on our close examination, we finally focused only on the colors of the board, from where we came up with a conjecture and proof.

 First, we established the checkerboard has 64 squares and requires 32 dominos to cover it completely. Knowing basic probability, we knew there are infinitely many ways the dominos could be arranged to cover the checkerboard but that was not the final goal. We started with mutilating the checkerboard by removing one corner square (or a single square), which was clearly impossible to cover it exactly. Then we removed two adjacent squares in one corner; like removing one domino. With this scenario the remaining squares were able to be covered by the dominos because it evened out the squares and was exactly one domino.



**Figure 3:** Mutilated checkerboard (Example1)

 Without any trivial cases, we asked ourselves: “Is it possible to cover the board when two opposite corners are removed?” we initially assumed that it is possible because there are 62 squares left on the board. Since 62 is an even number and 2 is an even number, based on the squares of a domino it would seem to work. We tried to place the dominos on the board, however our first attempt failed, then the second, and the third. Every time we tried to fit the dominos on the mutilated checkerboard, it left us with two squares uncovered, that turned out to be the same color. Unfortunately, our intuitive guess was incorrect; however we did learn that the even or odd theory did not directly link with the success of covering the board.



**Figure 4:** Mutilated checkerboard (Example 2)

Thenwe moved on to the second example, where we remove two corners (the top right and the bottom right corners) trying to figure out if it possible to completely cover the checkerboard with dominos. Without assuming anything, this time we went hands-on experimenting by drawing imaginary dominos around the checkerboard, some vertically and horizontally. And yes! This time it worked, we were able to cover the checkerboard completely with ease. Since we had already ruled out odd or even as being proof to solving the puzzle, we now began to check symmetry and if that impacted the results of the mutilated board. However the first and second examples were both symmetric and the results were totally different, then the symmetry hypothesis was ruled out.

 Having no clue to the reason behind the results of the examples mentioned above the only way we can find out was by creating more puzzles and testing them along with new proofs. We began to look closer at the second example noticing that the two corners that were removed were both black and white whereas, in the first example the two squares that were removed were either black or white. Therefore; the colors of the squares were the only thing that might have some relation to the success of covering the board. In other words, for the first example if you remove two white squares from the opposite corners and try to cover it with dominos then it will leave with two black uncovered squares. Similarly, the board cannot be covered if the same colored squares are removed because none of the dominos contains the same color. We thought this was just amazing! We were not 100% sold on the color theory yet, so we manipulated the board in different ways to ensure the certainty of this discovery.

 For instance, we randomly removed any two squares from the checkerboard and checked whether or not the board could be covered when the two squares were either from the same or opposite colors. The result was exactly the same as what we predicted. It became apparent to us that in order to cover the mutilated board with dominos, it is necessary to have the same number of blacks and white on the board, which means the amount of black squares taken out should be equal to the white ones, and vice versa.

 We also came up with a word problem as a representation to the game puzzle. Use example 1 as a hint to solve the problem. This is given as follows:

*“There are 32 students in a711 classroom and another 32 students in 712 classrooms. Mr. Reitz teaches Math for both 711 and 712. During math period he usually combines both classes together to do a pair-problem activity strictly between students of 711 and 712. One day, there were two students absent from class 711. Can Mr. Reitz through some arrangement make 31 successful pairs of 711 and 712 classes among 62 students?”*

Solution: Yes, Mr. Reitz can form 31 pairs between 711 and 712 classes. We notice that there is an abstract resemblance between this problem and example 1. According to this problem Mr. Reitz wants to match up students strictly between 711 and 712. So to solve this problem we want to math each item in one collection to an item in the other thereby making 31 pairs with two students from 712 left without their pair.

 Our new findings did not stop at this stage; we worked even further by viewing the checkerboard as a coordinate plane. We numbered up the board and across the bottom of the board to connect with the colors. We see that the coordinates of all the white colored squares have the same parity and the coordinates of the black squares are from different parities.

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 1 2 3 4 5 6 7 8

 Therefore, we stated our conjecture as the following; if the coordinates of the two removed squares both have the same parities, the board cannot be covered. For example: the two removed squares are (1, 3), and (3, 5). And if one of the squares has the same coordinates of the same parity and the other one is different parity, then the board can be covered. For example: the coordinates of the two removed squares are (1, 5) and (4, 3). The first coordinates are both odd and the second coordinates are odd and even meaning the board can be covered.

We created two puzzles to prove our conjecture is true under the given conditions of parity. To do this we will indicate which squares must be removed by giving the coordinates on the checkerboard as ordered pairs (x, y).

* **Puzzle 1:** Remove (4, 2) and (3, 5). Check if the checkerboard can be covered by dominos.

First, let’s see what colors of the squares these coordinates represent in the checkerboard.

 The coordinates represents two white squares and according to the conjecture the dominos cannot be covered completely. Let’s see if this true! (We used double arrows to represent dominos)



**Figure 6:** Incomplete covering of checkerboard

 Indeed! It is true that the dominos cannot cover the checkerboard completely when two squares of the same color are removed in this case it is two white squares. This also leaves us with two black squares because originally the numbers of black and white squares of the mutilated checkerboard are not equal. Therefore our conjecture is true.

* **Puzzle 2:** Remove (7, 4) and (2, 6). Can the checkerboard be covered?

**Figure 7:** Completed covering of checkerboard

Yes, again! Our conjecture is true. This time the dominos were able to cover the checkerboard because the two squares that were removed were black and a white. We know there are 32 pairs of black and white squares in other words we need 32 dominos to cover the checkerboards completely. So now, when we take away one pair i.e. one black and one white square we have 31 pairs left, which means that there are 31 double arrows in figure 7.

 Another way to find out if the checkerboards can be covered is that if the same number of black squares doesn’t equal to the same number of white squares then no matter what the case is, the dominos will not cover the whole checkerboard completely. This was an important observation because we had many trial and error cases where we tried to disprove this. But none of them seemed to work because they all went against what we had stated. Therefore, our conjecture that our group came up with is definitely true!