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MAT 2071/Sec D

Professor Reitz

The Mutilated Checkerboard Puzzle

The puzzle we worked on is called the Mutilated Checkerboard. The goal of this puzzle is to fill a regular 8x8 checkerboard with 1x2 dominos after removing two random squares from the checkerboard. The rules for this puzzle was that the dominos can be arranged either vertically or horizontally where the dominos cannot be placed outside of the checkerboard and cannot overlap each other. A hint that was given to us was that one domino piece can fit only two squares of the checkerboard, which made the puzzle fun and challenging.

When the professor first proposed this puzzle in class, as a group we thought that this puzzle would be simple because we knew that the dominos can fit into a checkerboard because a checkerboard has 64 squares and a domino is able to cover the two squares from the checkerboard. Once we knew this we saw that since the dominos is an even number and the checkerboard is an even number and there was no squares removed then the dominos was able to cover the whole checkerboard. However once we had our first challenge where we had to remove two squares and see if the dominos were able to fit into the checkerboard or not then this became hard. This puzzle was more challenging than what we were expecting because there was so many cases where we had to remove two different squares of the checkerboard and the result from this showed us that with some cases we tested the dominos wasn’t able to fit into the checkerboard and the other cases we tested the dominos were able to fit the whole checkerboard.

As we worked in class figuring out if the dominos was able to cover the checkerboard the puzzle became trickier and we came across two main cases. The first case is being that the dominos wasn’t able to cover the checkerboard. The second case was where the dominos was able to cover the checkerboard. These two cases helped us determined which square pieces can be removed and have the dominos cover the checkerboard. For example, for the first case we saw that by removing one square from any of the corner, it was impossible to cover the checkerboard with dominos. It was an exceedingly tedious process because of all the different arrangements. We tried different ways of arranging the dominos but we always end up with one square left over, which concluded that the dominos couldn’t fit. We have also experienced that by removing two tiles from the opposite corner sides (squares located on the same diagonal), the puzzle cannot be covered with dominos. For example we removed 1x1 square and 8x8 square the dominos will not fit, no matter what way we placed the dominos there would always be a square left over. For the second main case, we saw that the dominos was able to cover the checkerboard. We knew we can cover the checkerboard by removing two squares from the corners on the same side (those are two squares with the opposite colors). For example, we removed squares 1x1 square (white square) and we removed 1x8 (black square) the domino pieces were able to cover the checkerboard. This was one of the possible solutions where we saw that the dominos can cover the checkerboard.

Once we saw this, we then realized that when we placed a domino onto the checkerboard we saw that a domino always covers a black and white square. Knowing this concluded us to believe that when two of the same color squares were removed the dominos would not cover the checkerboard due to the fact that there will always be a square left over and because a domino can only cover a black and white square. This discovery made us wonder if the puzzle can always be solved by removing two squares of different colors from the checkerboard. However, we weren’t sure of this because there were so many choices when removing two squares and there was many possibilities of whether or not the dominos can cover the squares or not.

Our group developed a conjecture. The conjecture that we decided to work on was is if it was possible to fit domino pieces onto the checkerboard if two diagonal square pieces were removed or will it always be impossible to cover the checkerboard with the dominos if the squares of the same color are removed? To figure this conjecture out, we did a series of test to prove whether this statement was true or false. As we were working on the conjecture we first thought about how many ways you can remove two squares diagonal. This was hard because we saw that there were many ways to remove the squares so it became frustrating and tricky. Then we thought about if the diagonal pieces had to be next to each other or can be anywhere on the board as long as they were diagonal. So instead we decided to test them all out.

Since our group didn’t have time to see each other, we decided to test this conjecture out individually. Some of the tests we tried was removing odd and even squares, removing same of the colored squares, removing two different color squares, and removing squares from the middle of the checkerboard. We tested these out to see if we were able to cover the checkerboard with dominos when removing squares that were diagonally together and squares that were diagonally separated from each other. This took a long time to get because there was so many examples we tested and there was some stuff we were stuck on, so it was not easy to do this alone.

When removing two squares that were together diagonally, we came to a conclusion that it is not possible for the dominos to cover the checkerboard because there would always be two diagonal squares left over. When we tested this out we realized that the two diagonal that were together had the same color and by this we saw that the dominos was not able to cover the checkerboard. For example, we tried this by removing squares (3,2) and (4,3), which are two diagonal black squares, we saw that the dominos was unable to cover the checkerboard.

The next example we tested was when we removed two separated diagonal squares. We tested this out by using two different colored squares and also tested this by using two of the same colored squares. As we tested this example we surprising saw that when we have two different colored squares removed, the dominos was able to cover the checkerboard. However when we tested the example when removing two of the same colored dominoes in separate diagonals we saw that the dominos was unable to cover the checker board because there was always to squares left over. Again, this gave us an idea that whenever two different colored squares was removed from the checkerboard, the dominos will always cover it. But, this did not satisfy us yet because we still had a couple of example to test out to see if our conjecture was true or not.

So instead we tried removing two squares that were from the same diagonal and we tested this out when the diagonals were together and when they were separated and noticing the pattern we were able to conclude that it was impossible to cover the dominos in the checkerboard when two squares of the same color that are in the same diagonal are removed.

This led to an important discovery, which is, if you remove two of the same color squares whether it is two black or two white square pieces then the dominos will not cover the checkerboard because if you look at the checkerboard you will see that the checker board continuously alternates between white and black squares. As mentioned before, when you have a domino you already know that a domino can only cover two square pieces, and when you place one domino on the checkerboard you will see that the domino is meant to cover a black and a white piece only. If you remove two of the same color square pieces the dominos will not fit because there will be two extra square pieces left over. Concluding that our conjecture works if and only if there are two different colored square pieces removed. This conjecture is false when you remove two of the same colored square pieces because there will always be extra square pieces left over.