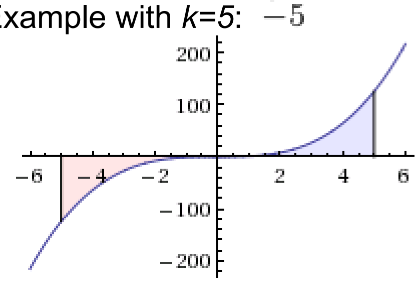
Reitz, CityTech
MAT1575

Exam 3 Review

Exam 3 will cover sections 7.4, 7.5, 7.8 and 8.1 The answer key follows the problems. If you find any errors or have any questions, please post them on the OpenLab or send me an email: jreitz@citytech.cuny.edu

1. A rational function is a fraction in which the numerator and denominator are both polynomials.
    a. Is the derivative of a rational function always a rational function? If so, how do you know? If not, give two examples of rational functions whose derivatives are not rational functions.
    b. Is the integral (antiderivative) of a rational function always a rational function? If so, how do you know? If not, give two examples of rational functions whose integrals are not rational functions.
2. The function $f(x)=x^3$ has what we call *odd symmetry*.
   FACT: If we integrate this function from -*k* to *k* for any real number *k*, we will always get zero, because the negative part (on the left) exactly cancels out the positive part (on the right). *The diagram below shows an example with* $k=5$ *.* QUESTION: Is the fact still true if we use $k=\infty$? That is, if we integrate $f(x)=x^3$ from $-\infty$ to ∞, do we get zero? Why or why not?

Example with *k*=*5*: $\displaystyle\int_{-5}^{5} x^3 dx = 0$



3. Give the form of the partial fraction expansion (you do NOT have to solve for A, B, C etc.):

   a. $\dfrac{6x-5}{x^2-5x-6}$ b. $\dfrac{3x+2}{x^3+6x}$ c. $\dfrac{1}{(x^2+3x+4)^3}$ d. $\dfrac{5x^3-11x^2+3x-7}{x(x^2+1)^2(3x+4)^3(x^2+2x+5)}$

4. Integrate: a. $\int \frac{x^3-7x+7}{x-2}dx$ b. $\int \frac{1}{x^2+12x+45}dx$

c. $\int \frac{3x^2-3x+10}{x^3+10x}dx$ d. $\int \frac{8x+32}{x^2+2x-15}dx$

5. Integrate: a. $\int \tan^2\theta \cdot \csc^2\theta \cdot \sin\theta d\theta$ b. $\int \frac{\sqrt{\ln x}}{x}dx$

c. $\int_1^4 (2+\sqrt{x})^9 dx$ d. $\int \frac{x^2}{\sec x}dx$

e. $\int \tan^3 x \sec^4 x dx$ f. $\int \frac{x^5}{\sqrt{1-x^2}}dx$

6. Determine whether each integral is convergent or divergent. If it is convergent, evaluate it.

a. $\int_1^\infty \frac{3}{x^5}dx$ b. $\int_{-\infty}^{-1} \frac{1}{\sqrt{1-x}}dx$ c. $\int_{-\infty}^{\infty} xe^{-x^2}dx$

d. $\int_2^3 \frac{1}{\sqrt[3]{2-x}}dx$ e. $\int_{-3}^{6} x^{-4}dx$

7. Use the comparison test to determine whether the integral is convergent or divergent: a. $\int_1^\infty e^{-x^4}dx$ b. $\int_5^\infty \frac{3+\sin^2 x}{x}dx$

8. a. Set up, but do not evaluate, an integral that represents the arc length of the function $f(x)=\frac{e^x}{x}$ on the interval $[-3,17]$.

b. Find the arc length of the function $y=(x-5)^{\frac{3}{2}}$ between $x=10$ and $x=20$.

Answer Key

1. *Here are some hints: for part a, what rule do we use to take the derivative of a rational function? What kind of function will that rule produce? For part b, think of some examples.*
2. *Hint: How do evaluate integrals from* $-\infty$ *to* ∞*? Try this and see what you get.*
3. a. $\frac{6x-5}{(x+1)(x-6)}=\frac{A}{x+1}+\frac{B}{x-6}$ b. $\frac{3x+2}{x(x^2+6)}=\frac{A}{x}+\frac{Bx+C}{x^2+6}$

   c. $\frac{1}{(x^2+3x+4)^3}=\frac{Ax+B}{x^2+3x+4}+\frac{Cx+D}{(x^2+3x+4)^2}+\frac{Ex+F}{(x^2+3x+4)^3}$

   d. $\frac{5x^3-11x^2+3x-7}{x(x^2+1)^2(3x+4)^3(x^2+2x+5)}=$

   $\frac{A}{x}+\frac{Bx+C}{x^2+1}+\frac{Dx+E}{(x^2+1)^2}+\frac{F}{3x+4}+\frac{G}{(3x+4)^2}+\frac{H}{(3x+4)^3}+\frac{Ix+J}{x^2+2x+5}$

4. a. $\int x^2+2x-3+\frac{1}{x-2}dx=\frac{x^3}{3}+x^2-3x+\ln|x-2|+C$ *(long division)*

   b. $\frac{1}{3}\arctan\frac{x+6}{3}+C$ *(completing the square)*

   c. $\int \frac{1}{x}+\frac{2x-3}{x^2+10}dx=\ln|x|+\ln|x^2+10|-\frac{3}{\sqrt{10}}\arctan\frac{x}{\sqrt{10}}+C$ *(partial fractions)*

   d. $\int \frac{1}{x+5}+\frac{7}{x-3}dx=\ln|x+5|+7\ln|x-3|+C$ *(partial fractions)*

5. a. $\sec\theta+C$ b. $\frac{2}{3}(\ln x)^{\frac{3}{2}}+C$ c. $\frac{18401976}{55}$

   d. $x^2\sin x+2x\cos x-2\sin x+C$ e. $\frac{1}{6}\tan^6 x+\frac{1}{4}\tan^4 x+C$

   f. $-\sqrt{1-x^2}+\frac{2}{3}(\sqrt{1-x^2})^3-\frac{1}{5}(\sqrt{1-x^2})^5+C$

   HINTS: a. change everything to sines and cosines, then substitute $u=\cos\theta$.
   b. substitute $u=\ln x$
   c. substitute $u=2+\sqrt{x}$ *(note: this also gives* $\sqrt{x}=u-2$*)*
   d. move $\sec x$ *to the top, then use integration by parts*
   e. substitute $u=\tan x$ *(convert two of the secants into tangents)*
   f. trigonometric substitution $x=\sin\theta$*, simplify, then save one sine and convert*

the rest to cosines

6. a. $\frac{3}{4}$ b. divergent c. 0 d. $-\frac{3}{2}$ e. divergent

7. a. Convergent. First rewrite $e^{-x^4}=\frac{1}{e^{x^4}}$, then compare $\frac{1}{e^{x^4}}<\frac{1}{e^x}$. The first function is smaller than the second, since the denominator is larger, and $\int_1^{\infty}\frac{1}{e^x}dx$ converges.

b. Divergent. Compare $\frac{3+\sin^2 x}{x}>\frac{3}{x}$. The first function is larger, since the denominator is larger (note that $\sin^2 x$ is always greater or equal to 0). The second function diverges when we integrate from 5 to ∞.

8. a. $\int_{-3}^{17}\sqrt{1+\left(\frac{xe^x-e^x}{x^2}\right)^2}dx$

b. $\int_{10}^{20}\sqrt{1+\left(\frac{3}{2}(x-5)^{\frac{1}{2}}\right)^2}dx=\frac{1}{27}(139\sqrt{139}-343)\approx 47.9921$