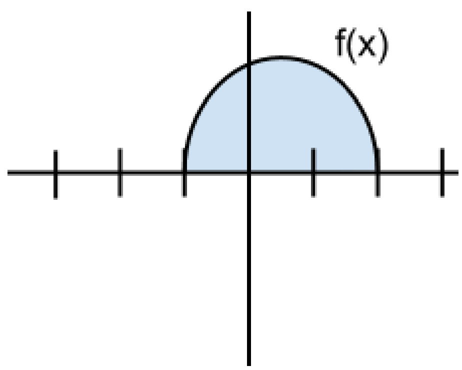
Exam 2 Review

The answer key follows the problems. If you find any errors or have any questions, please post them on the OpenLab or send me an email: jreitz@citytech.cuny.edu

1. We can calculate the area between two functions by using an integral (we integrate the "top function minus the bottom function"). When we do this, do we get an exact area, or an approximate area? Explain your answer.
2. Find the volume of the solid obtained by rotating the region lying between $y = e^x$ and the x-axis on the interval $[0,1]$ about the x-axis.
3. Use cylindrical shells to find the volume of the solid obtained by rotating about the y-axis the region enclosed by $y = -2x^2 + 6x$ and $y = 0$.
4. Find the volume of the solid obtained by rotating the region enclosed by $y = x^2 - 4$ and $y = 4 - x^2$ about the line $y = -5$.
5. Consider the volume obtained by rotating the region in the first quadrant bounded by $y = x^3$ and $y = x$ about the y-axis.
   a. Find the volume using cylindrical shells.
   b. Find the volume using washers.
6. Find the volume of the solid obtained by rotating about the y-axis the region under the function $y = \sin(\pi x^2)$ on the interval $[0,1]$
7. A bundt cake (shown below left) has a hole in the center 3 inches across. The cake can be obtained by rotating the region under the function $f(x)$ (shown below right) about an appropriate axis. Set up (but do not evaluate) an integral giving the volume of a cake made in this pan.



8. Evaluate: a. $\int x \cos x dx$ b. $\int_1^e x^3 \ln x \, dx$

c. $\int x^3 e^x dx$ d. $\int e^x \sin x dx$

9. Evaluate: a. $\int \sin x \cos^6 x dx$ b. $\int \sin^3 x \cos^4 x dx$

c. $\int \tan^4 x \sec^2 x dx$ d. $\int \sec^7 x \tan^3 x dx$

e. $\int_0^1 \cos^2(\pi x) dx$ f. $\int \sin(5x)\cos(9x) dx$ g. $\int \sec^3 dx$

10. The equation $x^2 + y^2 = 9$ describes a circle of radius 3 centered at the origin. Use calculus to find the area of the circle.

11. Evaluate: a. $\int \frac{1}{x^2\sqrt{36 - x^2}} dx$ b. $\int_{\sqrt{2}}^{2} \frac{1}{x^3\sqrt{x^2 - 1}} dx$

c. $\int \frac{dx}{\sqrt{x^2 + 25}}$

THINGS YOU MUST KNOW BY HEART:

Antiderivatives of:
$\sin x$, $\cos x$, $\tan x$, $\sec x$

Trig identities:
$\sin^2 x + \cos^2 x = 1$, $\tan^2 x + 1 = \sec^2 x$, $1 + \cot^2 x = \csc^2 x$

Answer Key

1. *I'd like you to answer in your own words, but I will provide a little direction (so you know what I'm interested in): the discussion of integrals generally begins by breaking up a shape up into approximating rectangles. Do the rectangles give the exact area? Does the integral give us the exact area? What's the connection between the rectangles and the integral?*
   (NOTE: In this question, I'm NOT interested in the fact that area is positive, but integrals can be negative -- I'm really interested in exploring the question of whether integrals can give the exact, or only an approximate, area of a shape).
2. $\int_0^1 \pi(e^x)^2 dx = \frac{\pi}{2}(e^2-1) \approx 10.036$
3. $\int_0^3 2\pi x(-2x^2+6x)dx = 27\pi \approx 84.823$
4. $\int_{-2}^{2} \pi\left[((4-x^2)+5)^2-((x^2-4)+5)^2\right]dx = \frac{640\pi}{3} \approx 670.206$
5. a. $\int_0^1 2\pi x(x-x^3)dx = \frac{4\pi}{15} \approx 0.838$ b. $\int_0^1 \pi((\sqrt[3]{y})^2-y^2)dy = \frac{4\pi}{15} \approx 0.838$
6. $\int_0^1 2\pi x \sin(\pi x^2)dx = 2$ *HINT: use u-substitution to evaluate!*
7. $\int_{-1}^{2} 2\pi(x+2.5)f(x)dx$
   HINT:Where should the axis of rotation be to ensure the hole in the center is the correct size?
8. a. $x \sin x + \cos x + C$ b. $\frac{3e^4+1}{16} \approx 10.300$ c. $e^x(x^3-3x^2+6x-6)+C$
   d. $\frac{1}{2}e^x(\sin x - \cos x)+C$
9. a. $-\frac{1}{7}\cos^7 x + C$ b. $-\frac{1}{5}\cos^5 x + \frac{1}{7}\cos^7 x + C$ c. $\frac{1}{5}\tan^5 x + C$
   d. $\frac{1}{9}\sec^9 x - \frac{1}{7}\sec^7 x + C$ e. $\frac{1}{2}$ f. $\frac{1}{8}\cos(-4x) - \frac{1}{28}\cos(14x) + C$
   g. $\frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + C$

10. $2\int_{-3}^{3}\sqrt{9-x^2}dx=9\pi\approx 28.274$

11. a. $-\frac{\sqrt{36-x^2}}{36x}+C$ b. $\frac{\pi+3\sqrt{3}-6}{24}\approx 0.0974$ c. $\ln\left|\frac{1}{5}\sqrt{x^2+25}+\frac{1}{5}x\right|+C$