Exam 1 Review

The answer key follows the problems. If you find any errors or have any questions, please post them on the OpenLab or send me an email: jreitz@citytech.cuny.edu

1. Find the derivative:
   a. $g(x)=\int_5^x t^4+3t^2+2t-1dt$
   b. $\int_{-3}^{x^3} \sin t \cos t\, dt$
   c. $\int_1^{\tan x} \sqrt{1+t}\, dt$
2. Find the indefinite integral:
   a. $\int \frac{x^3+x+\sqrt{x}}{x^2}dx$
   b. $\int x^3\sqrt{5-x^4}\, dx$
   c. $\int \frac{\sec^2 x}{\sqrt{tan\, x}}dx$
   d. $\int \frac{\ln x}{x}dx$
3. Evaluate the definite integral:
   a. $\int_0^{64} \sqrt[3]{x}(x-\sqrt{x})dx$
   b. $\int_{\frac{3\pi}{2}}^{\frac{\pi}{2}} \cos x\, dx$
   c. $\int_1^4 \frac{x^2}{1+x^3}dx$
   d. $\int_0^3 \sqrt{5x+1}\, dx$
4. At six o'clock in the morning a man enters his car, which is parked in his driveway. He drives in a straight line away from his home, accelerating at a rate given by $a(t)=25+\sin \pi t\ mi/hr^2$. At four o'clock that afternoon, where is the man and how fast is he going?
5. Consider the slogan: *"The definite integral gives the area."* In what ways it is accurate? In what ways is it inaccurate? Describe the relationship between definite integrals and areas.
6. Find the area: a. between $y=\sin x$ and $y=2-\sin x$ from $x=\frac{\pi}{2}$ to $x=\frac{3\pi}{2}$.
   b. bounded by $y=x^2-4$ and $y=4x-2x^2$ on the interval $[1,4]$
   c. bounded by the curves $y^2+3y=x$ and $5y=x-3$.
   d. enclosed by the graphs of $y=xe^{x^2}$ and $y=e\cdot x$

7. The graph of f(x), shown below, consists of a line segment and a semicircle. Use the graph to evaluate the definite integrals:

a. $\int_{0}^{4} f(x)dx$

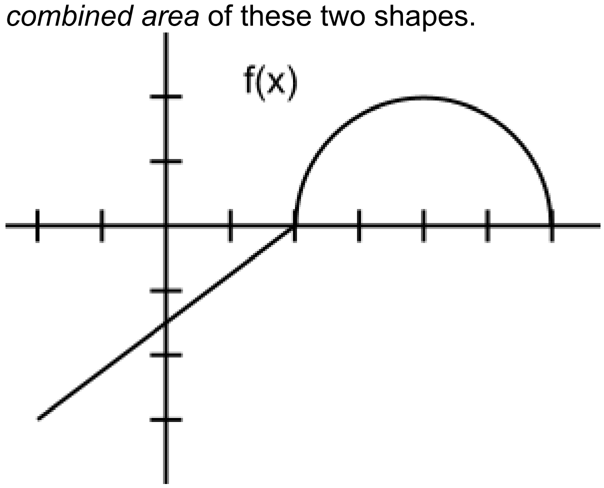
b. $\int_{-2}^{6} f(x)dx$

c. $\int_{-2}^{2} f(x)dx$

d. $\int_{2}^{-2} f(x)dx$

e. $\int_{-2}^{2} f(x)dx - \int_{2}^{6} f(x)dx$

f. In calculating the integrals above, you worked with two shapes -- a triangle and a semicircle. Write an expression (using one or more definite integrals) that gives the *combined area* of these two shapes.



Answer Key

1. a. $g'(x)=x^4+3x^2+2x-1$ b. $3x^2\sin(x^3)\cos(x^3)$
   c. $\sec^2 x\sqrt{1+\tan x}$
2. a. $\frac{1}{2}x^2+\ln|x|-\frac{2}{\sqrt{x}}+C$ b. $-\frac{1}{6}\sqrt{(5-x^4)^3}+C$
   c. $2\sqrt{\tan x}+C$ d. $\frac{1}{2}(\ln x)^2+C$
3. a. $\frac{454{,}656}{77}\approx 5904.62$ b. 2
   c. $=\frac{1}{3}\left(\ln\frac{65}{2}\right)\approx 1.16$ d. $\frac{42}{5}=8.4$
4. At 4 o'clock in the afternoon, the man is about 1253 miles from home (precise answer $1250+\frac{10}{\pi}$ miles), traveling 250 miles per hour.
   HINTS: we use the fact that $v(0)=0$ *and* $x(0)=0$ *to determine the velocity and positions functions:* $v(t)=25t-\frac{1}{\pi}\cos\pi t+\frac{1}{\pi}$, $x(t)=\frac{25}{2}t^2-\frac{1}{\pi^2}\sin\pi t+\frac{1}{\pi}t$, *and then plug t=10 into each function (t=10 gives 4 o'clock in the afternoon).*
5. *Answers will vary!*
6. a. $\displaystyle\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}(2-\sin x)-\sin x\,dx=2\pi$
   b. $\displaystyle\int_1^2(4x-2x^2)-(x^2-4)dx+\int_2^4(x^2-4)-(4x-2x^2)dx=3+24=27$
   c. $\displaystyle\int_{-1}^{3}(5y+3)-(y^2+3y)dy=\frac{32}{3}$
   HINTS: We integrate with respect to y. First find the bounds: solve each equation for x, then set them equal and solve to obtain the bounds y=-1 and y=3.
   d. $\displaystyle\int_{-1}^{0}xe^{x^2}-e\cdot x\,dx+\int_0^1 e\cdot x-xe^{x^2}dx=\frac{1}{2}+\frac{1}{2}=1$
   HINTS: Split each integral up into two integrals and deal with them separately, for

example $\int_0^1 e \cdot x dx - \int_0^1 x e^{x^2} dx$. *Use substitution to solve the second integral*

7. a. $\pi - \frac{3}{2}$ b. $2\pi - 6$ c. -6 d. 6 e. $-6 - 2\pi$

f. *There is more than one correct answer. The key idea here is that you will have to split up the integral into two parts, and in the first part (from -2 to 2) you will need to do something to change the sign from negative to positive. This could mean: reversing the limits of integration, or putting a negative sign in front of the integral, putting a negative sign in front of f(x), and so on. Here is one solution:*

$$Area = -\int_{-2}^{2} f(x)dx + \int_{2}^{6} f(x)dx$$