

Sample Exam III

MAT 1275 Spring 2012

Part I. Applications of Quadratic Equations.

1. The area of a rectangle is 60 square cm and the perimeter is 34 cm. Find the length and width of the rectangle.
2. Suppose that the length of one leg of a right triangle is 3 inches more than the length of the other leg. If the length of the hypotenuse is 15 inches, find the lengths of the two legs.
3. A right triangle has side lengths represented by three consecutive even integers. Find the lengths of the three sides, measured in meters.

Part II. Graphs of Quadratic Functions.

1. Given the following functions, write them in the form $f(x) = a(x-h)^2+k$ by completing the square then graph them. Make sure to identify the vertex, axis of symmetry, minimum function value and any (x) or (y)-intercepts.

(a) $g(x) = 2x^2 + 12x + 13$

(b) $h(x) = x^2 + 4x + 5$

2. Graph these equations. Label the coordinates of the vertex, and write the equation of the axis of symmetry.

(a) $y = \frac{1}{3}x^2 + 5$

(b) $y = (x + 5)^2 - 2$

(c) $y = 2x^2 + 8x + 9$

(d) $y = x^2 + 4x$

Part III. Distance Formula, Midpoint and Circles and Perpendicular Bisector

1. Find the radius of a circle with endpoints of a diameter $(-2, 3)$ and $(4, 1)$

2. Identify the center and radius of the circle and then graph the circle. Complete the square if necessary.

(a) $(x - 3)^2 + (y + 1)^2 = 16$

(b) $(x + 1)^2 + y^2 = 1$

(c) $x^2 + y^2 + 4x - 8y + 16 = 0$

3. Find the equation of the perpendicular bisector of the line segment joining the pair of points $(0, 5)$ and $(4, -5)$.

Part IV. Systems of Equations.

Solve these systems of equations. If there is not a unique solution, label the system as either dependent or inconsistent.

1.

$$\begin{aligned}x + 2y - 3z &= 2 \\-2x + y + 2z &= 12 \\3x - 4y + z &= -24\end{aligned}$$

2.

$$\begin{aligned}x + y &= z \\2x + 4y - 2z &= 6 \\3x + 6y - 3z &= 9\end{aligned}$$

3.

$$\begin{aligned}3x + 2y + z &= 3 \\x - 3y + z &= 4 \\-6x - 4y - 2z &= 1\end{aligned}$$

4.

$$\begin{aligned}x^2 + xy &= 7 \\x + 2y &= 5\end{aligned}$$

5.

$$3x^2 + 4y^2 = 16$$

$$2x^2 - 3y^2 = 5$$

Solutions:

Part I.

1. length= 5cm, width=12cm, or width=5cm and length=12cm
2. one leg=9 in, the other 12 in
3. the lengths are 6m, 8m and 10m

Part II.

1. (a) vertex $(-3, -5)$, axis of symmetry $x = -3$, no minimum, maximum at $y = -5$, no (x)-intercepts, (y)-intercept at $(0, 13)$
(b) vertex $(-2, 1)$, axis of symmetry $x = -2$, minimum value at $y = 1$, no maximum, no (x)-intercepts, (y)-intercept at $(0, 5)$
2. (a) vertex $(0, 5)$, axis of symmetry $x = 0$
(b) vertex $(-5, -2)$, axis of symmetry $x = -5$
(c) vertex $(-2, 1)$, axis of symmetry $x = -2$
(d) vertex $(-2, -4)$, axis of symmetry $x = -2$

Part III.

1. radius= $\sqrt{10}$
2. (a) center $(3, -1)$, radius=4
(b) center $(-1, 0)$, radius=1
(c) center $(-2, 4)$, radius=2

3. $y = \frac{2}{5}x - \frac{4}{5}$

Part IV.

1. $(-3, 4, 1)$

2. dependent system

3. inconsistent system

4. $(2, \frac{3}{2}), (-7, 6)$

5. $(2, 1), (-2, 1), (2, -1), (-2, -1)$