1. Evaluate each determinant. Show complete work.

(a) $\begin{vmatrix} 3 & -2 & 1 \\ 3 & -1 & -2 \\ 3 & -2 & -3 \end{vmatrix}$

(b) $\begin{vmatrix} 3 & 4 & 5 \\ -4 & 6 & 3 \\ 1 & -4 & 3 \end{vmatrix}$

(c) $\begin{vmatrix} 0 & a & b \\ 0 & c & d \\ 0 & x & y \end{vmatrix}$

2. What value of x makes the determinant -4?

$$\begin{vmatrix} -2 & 0 & 0 \\ -6 & x & 1 \\ -4 & 0 & -1 \end{vmatrix}$$

3. Find whether the following matrix is invertible or not. Singular or non-singular? **Do not find the inverse.** Show your work.

$$\begin{vmatrix} 2 & 4 & 1 & 2 \\ -3 & 0 & 0 & 0 \\ 1 & 3 & 2 & 1 \\ -2 & 9 & 3 & 1 \end{vmatrix}$$

4. Let A and B be 4 x 4 matrices with det(A) = -1 and det(B)=2. Then compute

(a) det(AB)

(b) $\det(B^5)$

(c) det(2A)

(d) $\det(A^T A)$

(e) $\det(B^{-1}AB)$

(f) $\det(AB)^T$

5. True or False. Explain your answer mathematically.
   (a) If A is invertible and 1 is an eigenvalue for A, then 1 is an eigenvalue for A^{-1}.

   (b) If A is row equivalent to the identity matrix then A is diagonalizable.

   (c) Each eigenvalue of A is also an eigenvalue of A^2.

   (d) Two eigenvectors corresponding to the same eigenvalue are always linearly dependent.

   (e) Similar matrices always have the same eigenvalues.

   (f) Similar matrices always have the same eigenvectors.

   (g) If A is nxn diagonalizable matrix, then each vector in R^n can be written as a linear combination of eigenvectors of A.

6. Find the eigenvalues of the following matrices.

   (a) $\begin{bmatrix} 3 & -2 & 8 \\ 0 & 5 & -2 \\ 0 & -4 & 3 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 5 & -6 & -7 \\ 2 & 4 & 5 & 2 \\ 0 & 0 & -7 & -4 \\ 0 & 0 & 3 & 1 \end{bmatrix}$

7. Let $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$. Show that the characteristic polynomial of A is

$$\lambda^2 - (\text{trace } A)\lambda + \det(A).$$

8. Compute A^8 when $A = \begin{pmatrix} 4 & -3 \\ 2 & -1 \end{pmatrix}$.

9. Let

$$A = \begin{pmatrix} 1 & -3 & -3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$$

Eigenvalues $\lambda = 4, -2$ (repeated)

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \vec{v}_3 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$\vec{v}_1, \vec{v}_2, \vec{v}_3$ are the eigenvectors corresponding to the eigenvalues $4, -2, -2$ respectively

Use this information to diagonalize A.

10. Diagonalize the following matrices, if possible.

(a) $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$ (b) $B = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -2 \\ 1 & 3 & 1 \end{bmatrix}$