

Topics to be covered from Ch. 1.8, 2.1, 2.2

1. Consider the problem of determining whether the following system of equations is consistent.

$$4x_1 - 2x_2 + 7x_3 = -5$$

$$8x_1 - 3x_2 + 10x_3 = -3$$

- (a) Define appropriate vectors, and restate the problem in terms of linear combinations. Then solve the problem.  
 (b) Define an appropriate matrix, and restate the problem using the phrase “columns of A.”  
 (c) Define an appropriate linear transformation T using matrix (b), and restate the problem in terms of T.

2. Consider the problem of determining whether the following system of equations is consistent for all  $b_1$ ,  $b_2$ , and  $b_3$ .

$$2x_1 - 4x_2 - 2x_3 = b_1$$

$$-5x_1 + x_2 + x_3 = b_2$$

$$7x_1 - 5x_2 - 3x_3 = b_3$$

- (a) Define appropriate vectors, and restate the problem in terms of  $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ . Then solve the problem.  
 (b) Define an appropriate matrix, and restate the problem using the phrase “columns of A.”  
 (c) Define an appropriate linear transformation T using the matrix (b), and restate the problem in terms of T.

3. State whether the following statements are TRUE or FALSE. Justify your answer.

- (a) If A and B are  $m \times n$ , then both  $AB^T$  and  $A^TB$  are defined.  
 (b) If  $AB = C$  and C has 2 columns, then A has 2 columns.  
 (c) If  $BC=BD$ , then  $C=D$ .  
 (d) If A and B are  $n \times n$ , then  $(A+B)(A-B)=A^2 - B^2$ .

4. Let  $A = \begin{bmatrix} 1 & 3 & 8 \\ 2 & 4 & 11 \\ 1 & 2 & 5 \end{bmatrix}$  and  $B = \begin{bmatrix} -3 & 5 \\ 1 & 5 \\ 3 & 4 \end{bmatrix}$ .

- (a) Compute  $A^{-1}$ .  
 (b) Can you compute  $AB$  and  $BA$ ? Why or why not?  
 (c) Compute  $AB$  and  $A^TB$ .

5. Solve the following system of equations using matrix inversion. Find the inverse matrix manually and solve for  $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$

$$x + 4z = 2$$

$$x + y + 6z = 3$$

$$-3x - 10z = 4$$

6. Linear Transformations from  $\mathbb{R}^n$  to  $\mathbb{R}^m$ .

(a), (b) and (c) Which of the following are linear? Justify your conclusion.

(a)

$$g \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ x + y \end{pmatrix}$$

(b)

$$h \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} xy \\ x + y \end{pmatrix}$$

(c)

$$f \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z - x \\ x + y \end{pmatrix}$$

7. Suppose an  $n \times n$  matrix  $A$  satisfies the equation  $A^2 - 2A + I = 0$ . Show that  $A^3 = 3A - 2I$  and  $A^4 = 4A - 3I$

8. Find a matrix  $A$  such that the transformation  $x \rightarrow Ax$  maps  $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$  and  $\begin{bmatrix} 2 \\ 7 \end{bmatrix}$  into

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \text{ respectively}$$

9. Suppose  $A$ ,  $B$ , and  $X$  are  $n \times n$  matrices with  $A$ ,  $X$ , and  $A - AX$  invertible, and suppose  $(A - AX)^{-1} = X^{-1}B$ .

(a) Explain why  $B$  is invertible.

(b) Solve the equation for  $X$ . If a matrix needs to be inverted, explain why that matrix is invertible.