SEC 5.5 BINOMIAL RANDOM VARIABLES KING

P. 244 2, 3, 5, 10, 11, 21, 23

2. (a) $\frac{8!}{3!5!} = \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{3 \cdot 2 \cdot 1 \cdot \cancel{5!}} \quad \frac{336}{6} = 56$

(b) $\frac{7!}{3!4!} = \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{3 \cdot 2 \cdot 1 \cdot \cancel{4!}} = \frac{210}{6} = 35$

(c) $\frac{9!}{4!5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{4 \cdot 3 \cdot 2 \cdot 1 \cdot \cancel{5!}} = \frac{3024}{24} = 126$

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3. $9! = 362880$ FIND $10! = 3628800$

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5. X IS BINOMIAL $n = 8$ $p = 0.4$

(a) $P\{X = 3\}$

$= \frac{8!}{3!5!}(.4)^3(.6)^5$

$= \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{3 \cdot 2 \cdot 1 \cdot \cancel{5!}}(.064)(.07776)$

$= \frac{336}{6}(.064)(.07776)$

$= 56(.064)(.07776) = .27869184$

(b) $P\{X = 5\}$

$= \frac{8!}{5!3!}(.4)^5(.6)^3$

$= \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!}\,3 \cdot 2 \cdot 1}$

$= 56(.01024)(.216) = .12386304$

(c) $P\{X = 7\}$

$= \frac{8!}{7!1!}(.4)^7(.6)^1$

$= \frac{8 \cdot \cancel{7!}}{\cancel{7!}\,1!}$

$= 8(.0016384)(.6) = .00786432$

10. multiple choice 3 answers for 5 questions

$p = \frac{1}{3}$

$P\{X \geq 4\} = P\{X=4\} + P\{X=5\}$

$= \frac{5!}{4!\,1!} \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^1$

$= \frac{5 \cdot \cancel{4!}}{\cancel{4!}\,\cancel{1!}} (.012345679)(.66666667) = .0411522633$

$= \frac{\cancel{5!}}{\cancel{5!}\,0!} \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^0$

$= (1)(.0041152263)(1) = .0041152263$

DECIMAL

$\checkmark\ (.0452674896)$

$= \frac{5!}{4!\,1!} \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^1$

$= \frac{5 \cdot \cancel{4!}}{\cancel{4!}\,\cancel{1!}} \left(\frac{1}{81}\right)\left(\frac{2}{3}\right) = 5\left(\frac{2}{243}\right) = \frac{10}{243}$

$= \frac{5!}{5!\,0!} \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^0 = (1)\left(\frac{1}{243}\right)\left(\frac{1}{1}\right) = \frac{1}{243}$

$= \frac{11}{243}$ ✓

FRACTIONS

11. $n = 8$ $p = 0.5$ FAIR

6 CORRECT

$$P\{x \ge 6\} = P\{x=6\} + P\{x=7\} + P\{x=8\}$$

$$= \frac{8!}{6!\,2!}(.5)^6(.5)^2 = \frac{8\cdot 7\cdot \cancel{6!}}{\cancel{6!}\,2\cdot 1}(.015625)(.25)$$

$$= 28(.015625)(.25) = .109375$$

$$= \frac{8!}{7!\,1!}(.5)^7(.5)^1 = \frac{8\cdot \cancel{7!}}{\cancel{7!}\,1}(.0078125)(.5)$$

$$= 8(.0078125)(.5) = .03125$$

$$= \frac{8!}{8!\,0!}(.5)^8(.5)^0 = (1)(\underline{.00390625})(1)$$

ADD ALL THREE UP = .14453125 ✓

USING FRACTIONS

$$6 \quad = \frac{8!}{6!\,2!}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^2 = 28\left(\frac{1}{64}\right)\left(\frac{1}{4}\right) = \frac{28}{256}$$

$$7 \quad = \frac{8}{7!\,1!}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^1 = 8\left(\frac{1}{128}\right)\left(\frac{1}{2}\right) = \frac{8}{256}$$

$$8 \quad = \frac{8!}{8!\,0!}\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^0 = (1)\left(\frac{1}{256}\right)(1) = \frac{1}{256}$$

$$= \frac{37}{256}$$ ✓

21. FBI 44% KILLED w/ HANDGUNS
$n = 4$ VICTIMS SELECTED

ⓐ $P = \{x=4\}$ $\frac{4!}{4!\,0!}(.44)^4(.56)^0$
$(1)(\underline{\underline{.03748096}})(1)$

ⓑ $P = \{x=0\}$ $\frac{4!}{0!\,4!}(.44)^0(.56)^4$
$(1)(1)(\underline{.09834496})$

ⓒ $P = \{x \geq 2\} = 1 - P\{x \leq 1\}$ $(\cancel{P\{x=0\}\{x=1\}})$
$0 \quad 1 \quad \underline{2 \quad 3 \quad 4}$
$1 - P\{x=1\} + P\{x=0\}$

SAME

$P\{x=1\} = \frac{4!}{1!\,3!}(.44)^1(.56)^3$
$= \frac{4 \cdot 3!}{1!\,\cancel{3!}}(.44)(.175616) = .3090846$

$P\{x=0\} = \frac{\cancel{4!}}{0!\,\cancel{4!}}(.44)^0(.56)^4$
$(1)(1)(.09834496)$

ADD $\underline{.3090846 + .09834496}$
$1 - (.40742912) = .59257088$

ⓓ $E[x] = n \cdot p = 4 \cdot .44 = 1.76$

ⓔ $SD = \sqrt{n \cdot p \cdot q} = \sqrt{4 \cdot .44 \cdot .56}$
$= \sqrt{.9856} = .9927738917$

23. X BINOMIAL

$$E[X] = 4 = n \cdot p$$

$$\mathrm{VAR}(X) = n \cdot p(1-p) = 2.4$$

FIRST FIND $\underline{\underline{p}}$

$$2.4 = n \cdot p(1-p)$$

$$\frac{2.4}{p(1-p)} = n$$

$$4 = n \cdot p$$

$$4 = \frac{2.4}{\cancel{p}(1-p)} \cdot \cancel{p}$$

$$4(1-p) = 2.4$$

$$4 - 4p = 2.4$$

$$-4 \qquad -4$$

$$-4p = -1.6$$

$$p = .4$$

NOW FIND $\underline{\underline{n}}$

$$4 = n \cdot p$$

$$4 = n \cdot (.4)$$

$$10 = n$$

$$\boxed{p = .4 \quad n = 10}$$

NOW SOLVE

$P\{X=0\}$

$P\{X=12\} \leftarrow \varnothing$

23 (a) WE KNOW $p = .4$ $n = 10$

$$\binom{10}{0}$$

$$p\{x=0\} = \frac{10!}{0!\,10!}(.4)^0(.6)^{10}$$

$$= (1)(1)(.0060466176)$$

.00604662 (BOOK)

(b) $\binom{10}{12} = \emptyset$ NOT POSSIBLE IF $n = 10$

24. X BINOMIAL

$$E[X] = 4.5 = n \cdot p$$

$$VAR(X) = .45 = n \cdot p(1-p)$$

FIND n + FIND p FIRST.

$$.45 = n \cdot p(1-p)$$

$$\frac{.45}{p(1-p)} = n$$

$$4.5 = n \cdot p$$

$$4.5 = \frac{.45}{\cancel{p}(1-p)} \cdot \cancel{p}$$

$$4.5(1-p) = .45$$

$$4.5 - 4.5p = .45$$

$$-4.5 \quad -4.5$$

$$\frac{-4.5p}{-4.5} = \frac{-4.05}{4.5}$$

$$p = .9$$

$$4.5 = n \cdot p$$

$$\frac{4.5}{.9} = n \cdot .9 / .9$$

$$4.5/.9 = n = 5$$

$$n = 5 \qquad p = .9$$

NOW SOLVE LIKE ANY PROBLEM

24(a) WE KNOW $n=5$ $p=.9$

$P\{x=3\}$ $\binom{5}{3}$

$$\frac{5!}{3!\,2!}(.9)^3(.1)^2$$

$$\frac{5\cdot4\cdot\cancel{3!}}{\cancel{3!}\cdot2!}(.729)(.01)$$

$$(10)(.729)(.01)=\boxed{.0729}$$

(b) $P\{x\geq 4\} = P\{x4\} + P\{x=5\}$

$$P\{x=4\}=\frac{5!}{4!\,1!}(.9)^4(.1)^1$$

$$\frac{5\cdot\cancel{4!}}{\cancel{4!}\,1!}(.6561)(.1)=.32805$$

$$P\{x=5\}=\frac{\cancel{5!}}{\cancel{5!}\,\cancel{0}!}(.9)^5(.1)^0$$

$$(1)(.9)^5(1)=.59049$$

ADD THEM UP $\boxed{.91854}$