Soln.

MAT 2580 Quiz6 Name ______________________ Prof. Ghosh-Dastidar Date 5/9/13

Show all of your work. No points will be given if work is not shown. Good Luck!

1. **Diagonalize** A and find a formula for A^{10} **to diagonalize** A^{10} where $A = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}$.

No calculator should be used. Need to calculate the inverse by hand. (4+3+3+5+5)

$$A = \begin{pmatrix} 6 & -1 \\ 2 & 3 \end{pmatrix} \qquad |A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 6-\lambda & -1 \\ 2 & 3-\lambda \end{vmatrix} = 0 \Rightarrow (6-\lambda)(3-\lambda) + 2 = 0$$

$$\Rightarrow 18 - 3\lambda - 6\lambda + \lambda^2 + 2 = 0$$

$$\Rightarrow \lambda^2 - 9\lambda + 20 = 0$$

$$\Rightarrow (\lambda-5)(\lambda-4) = 0 \quad \boxed{\lambda = 4, 5} \quad 4$$

Eigenvectors: $\underline{\lambda_1 = 4}$

$$\begin{pmatrix} 6-4 & -1 \\ 2 & 3-4 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 2 & -1 \end{pmatrix} \xrightarrow{R_2 - R_1} \begin{pmatrix} 2 & -1 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow 2x_1 - x_2 = 0 \Rightarrow 2x_1 = x_2 \Rightarrow x_1 = \tfrac{1}{2}x_2, \quad x_2 \text{ free}$$

$$\vec{v}_1 \text{ (for } \lambda_1) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} \text{Taking} \\ x_2 = 2 \end{pmatrix} \quad 3$$

~~Eigen~~ $\underline{\text{For } \lambda_2 = 5}$

$$\begin{pmatrix} 6-5 & -1 \\ 2 & 3-5 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix} \xrightarrow{R_2 - 2R_1} \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \quad 3$$

$$x_1 - x_2 = 0 \Rightarrow x_1 = x_2, \quad x_2 \text{ free}.$$

$$\vec{v}_2 \text{ (associated with } \lambda_2 = 5) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{Taking } x_2 = 1$$

We know $A\vec{x} = \lambda\vec{x} \Rightarrow A(A\vec{x}) = A(\lambda\vec{x}) \Rightarrow$

$$\Rightarrow A^2\vec{x} = \lambda(A\vec{x}) \Rightarrow A^2\vec{x} = \lambda(\lambda\vec{x}) \Rightarrow A^2\vec{x} = \lambda^2\vec{x} \cdots\cdots A^k\vec{x} = \lambda^k\vec{x}$$

$\Rightarrow$ Eigenvectors stay same. Eigenvalues change to λ^k.

$\Rightarrow A^{10}\vec{x} = \lambda^{10}\vec{x}$

Eigenvalues for $A^{10} = 4^{10},\ 5^{10}$ 5

$\Rightarrow A^{10} = PDP^{-1} = \begin{pmatrix}1 & 1\\ 2 & 1\end{pmatrix}\begin{pmatrix}4^{10} & 0\\ 0 & 5^{10}\end{pmatrix}\begin{pmatrix}1 & 1\\ 2 & 1\end{pmatrix}^{-1}$

$\begin{pmatrix}1 & 1\\ 2 & 1\end{pmatrix}^{-1} = \frac{1}{1-2}\begin{pmatrix}1 & -1\\ -2 & 1\end{pmatrix} = \begin{pmatrix}-1 & 1\\ 2 & -1\end{pmatrix}$ 4

$$\Rightarrow \boxed{A^{10} = \begin{pmatrix}1 & 1\\ 2 & 1\end{pmatrix}\begin{pmatrix}4^{10} & 0\\ 0 & 5^{10}\end{pmatrix}\begin{pmatrix}-1 & 1\\ 2 & -1\end{pmatrix}}$$ 1