Solution

MAT 2580 Quiz5 Name ____________________ Prof. Ghosh-Dastidar Date 4/30/13

Show all of your work. No points will be given if work is not shown. Good Luck!

1. True or false: Justify your answer. Each eigenvalue of A is also an eigenvalue of A^2. (4)

FALSE

We learned in class:

$$\left.\begin{aligned} A\vec{x} &= \lambda\vec{x} \\ A(A\vec{x}) &= A(\lambda\vec{x}) \\ \Rightarrow A^2(\vec{x}) &= \lambda A\vec{x} \\ A^2(\vec{x}) &= \lambda(\lambda\vec{x}) \\ A^2\vec{x} &= \lambda^2\vec{x} \end{aligned}\right\} \Rightarrow$$

Eigenvalues of A^2 should be λ^2 where λ is eigenvalue of A.

2. Is $\begin{bmatrix}1\\-2\\2\end{bmatrix}$ an eigenvector of $\begin{bmatrix}3&6&7\\3&2&7\\5&6&4\end{bmatrix}$? If so, find the associated eigenvalue. (6)

We know $A\vec{x} = \lambda\vec{x} \Rightarrow$

$$\begin{bmatrix}3&6&7\\3&2&7\\5&6&4\end{bmatrix}\begin{bmatrix}1\\-2\\2\end{bmatrix} = \lambda\begin{bmatrix}1\\-2\\2\end{bmatrix}$$

$$\Rightarrow \begin{pmatrix}3-12+14\\3-4+14\\5-12+8\end{pmatrix} = \lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}5\\13\\1\end{pmatrix} = \lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}$$

If $\begin{pmatrix}1\\-2\\2\end{pmatrix}$ is an e-vector of A then it must satisfy $A\vec{x} = \lambda\vec{x}$. So can we find a scalar λ such that $A\vec{x} = \lambda\vec{x}$ where $\vec{x} = \begin{pmatrix}1\\-2\\2\end{pmatrix}$?

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So $\begin{pmatrix}1\\-2\\2\end{pmatrix}$ is <u>NOT</u> an eigenvector of the given matrix since we cannot find a scalar multiple λ that satisfies $A\vec{x} = \lambda\vec{x}$

3. Is $\lambda = 1$ an eigenvalue of $\begin{bmatrix} 4 & -2 & 3 \\ 0 & -1 & 3 \\ -1 & 2 & -2 \end{bmatrix}$. If so, find one corresponding eigenvector. (10)

$(A - \lambda I)\vec{x} = \vec{0}$ (if λ is an eigenvalue)

$$\Rightarrow \begin{pmatrix} 4-1 & -2 & 3 \\ 0 & -1-1 & 3 \\ -1 & 2 & -2-1 \end{pmatrix} = \begin{pmatrix} 3 & -2 & 3 \\ 0 & -2 & 3 \\ -1 & 2 & -3 \end{pmatrix}$$

$$\xrightarrow{\text{rref}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{3}{2} \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow x_2 - \frac{3}{2}x_3 = 0$$

$x_2 = \frac{3}{2}x_3$, x_3 free var.

$x_1 = 0$

$$\vec{v} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{3}{2}x_3 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3/2 \\ 1 \end{pmatrix} x_3 = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}, \ x_3$$

So the corresponding eigenvector is $\begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$