Exam #3 Review Solns. #2, #9.

2(a)
$$\begin{vmatrix} 12 & 13 & 14 \\ 15 & 16 & 17 \\ 18 & 19 & 20 \end{vmatrix} \overset{R_2-R_1}{\underset{R_3-R_1}{=}} \begin{vmatrix} 12 & 13 & 14 \\ 3 & 3 & 3 \\ 6 & 6 & 6 \end{vmatrix}$$

$$\underset{=}{R_3-2R_2} \begin{vmatrix} 12 & 13 & 14 \\ 3 & 3 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 0$$ since the whole last row $= 0$.

(since if we add or subtract a multiple of one row to another row, determinant stays same).

2(b)
$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{vmatrix} \overset{R_2-R_1}{\underset{R_3-R_1}{=}} \begin{vmatrix} 1 & a & b+c \\ 0 & b-a & a-b \\ 0 & c-a & a-c \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & b+c \\ 0 & b-a & -(b-a) \\ 0 & c-a & -(c-a) \end{vmatrix}$$

valid since determinant does not change for the reason mention in 2(a)

$$= (b-a)(c-a) \begin{vmatrix} 1 & a & b+c \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{vmatrix}$$

(since we know if we multiply one row of A by k to get B
$k\det(A) = \det(B)$

So here
$\det(A) = (b-a)(c-a)\det(B)$

#9.

$$A = \begin{pmatrix} 5 & 3 & 2 & -6 & -8 \\ 4 & 1 & 3 & -8 & -7 \\ 5 & 1 & 4 & 5 & 19 \\ -7 & -5 & -2 & 8 & 5 \end{pmatrix}$$

$$\xrightarrow{\text{rref}} \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

pivot columns.

$$\Rightarrow \left\{ \begin{pmatrix} 5 \\ 4 \\ 5 \\ -7 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 1 \\ -5 \end{pmatrix}, \begin{pmatrix} \cancel{2}\,-6 \\ -8 \\ 5 \\ 8 \end{pmatrix}, \begin{pmatrix} -8 \\ -7 \\ 19 \\ 5 \end{pmatrix} \right\}$$

form basis for the column space.

For null space

$X_5 = 0$, $X_4 = 0$, $X_2 - X_3 = 0 \Rightarrow X_2 = X_3$

$X_1 + X_3 = 0 \Rightarrow X_1 = -X_3$

X_3 free var.

$$\vec{x} = \begin{pmatrix} -X_3 \\ X_3 \\ X_3 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} X_3$$

Basis for Null space of $A = \left\{ \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$