Exam 2 Review

part IV solns. probs. 3 & 4.

#3. $\left\{ \begin{pmatrix} 1 \\ a \end{pmatrix}, \begin{pmatrix} a \\ a+2 \end{pmatrix} \right\}$ needs to be linearly indept.

$c_1 \begin{pmatrix} 1 \\ a \end{pmatrix} + c_2 \begin{pmatrix} a \\ a+2 \end{pmatrix} = 0$ should have trivial solns.

$\Rightarrow \left(\begin{array}{cc:c} 1 & a & 0 \\ a & a+2 & 0 \end{array} \right) \xrightarrow[R_2 - aR_1]{} \begin{pmatrix} 1 & a & 0 \\ 0 & (a+2)-a^2 & 0 \end{pmatrix}$

$\longrightarrow \begin{pmatrix} 1 & a & 0 \\ 0 & (a+2)-a^2 & 0 \end{pmatrix}$

For the eqn. to have only trivial soln.
we should not have free variables

$\Rightarrow a+2-a^2 = 0$

$\Rightarrow a^2 - a - 2 = 0$

$\Rightarrow (a-2)(a+1) = 0$

$\boxed{a = 2, \quad a = -1}$

Therefore for $a = 2, a = -1$ the eqn. will have non-trivial soln.

$\Rightarrow$ for all values of a except $a = 2, a = -1$ the vectors will be linearly indept. i.e, $a \neq 2, -1$.

#4 If $\vec{v}_4$ is not in span$\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ $\Rightarrow \vec{v}_4$ cannot be expressed in terms of $\vec{v}_1, \vec{v}_2$, and $\vec{v}_3$.
Also $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is given that linearly indept.
$\Rightarrow \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ must be linearly indept.