Ex#2 Review Soln. part III

#7.
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 1 & 0 & 0 \\ 1 & 1 & 6 & 0 & 1 & 0 \\ -3 & 0 & -10 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_3+3R_1]{R_2-R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 2 & 3 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2-R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & 1 & -1 \\ 0 & 0 & 2 & 3 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_1-2R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & 0 & -2 \\ 0 & 1 & 0 & -4 & 1 & -1 \\ 0 & 0 & 2 & 3 & 0 & 1 \end{array}\right]$$

$$\frac{1}{2}R_3 \left[\underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I} \;\middle|\; \underbrace{\begin{matrix} -5 & 0 & -2 \\ -4 & 1 & -1 \\ \frac{3}{2} & 0 & \frac{1}{2} \end{matrix}}_{A^{-1}}\right]$$

$$\Rightarrow \quad A^{-1} = \begin{pmatrix} -5 & 0 & -2 \\ -4 & 1 & -1 \\ \frac{3}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

Soln. $\vec{x} = A^{-1}\vec{b} = \begin{pmatrix} -5 & 0 & -2 \\ -4 & 1 & -1 \\ \frac{3}{2} & 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} -10-8 \\ -8+3-4 \\ 3+0+2 \end{pmatrix} = \begin{pmatrix} -18 \\ -9 \\ 5 \end{pmatrix}$

Soln.: $\boxed{x=-18,\ y=-9,\ z=5.}$

Checking

$$x + 4z = 2$$
$$-18 + 4(5) = 2$$
$$2 = 2 \checkmark$$

$$x + y + 6z = 3$$
$$-18 - 9 + (6)(5) = 3$$
$$-27 + 30 = 3$$
$$3 = 3 \checkmark$$

$$-3x - 10z = 4$$
$$-3(-18) - 10(5) = 4$$
$$+54 - 50 = 4$$
$$4 = 4 \checkmark$$

#8. (a) $g\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ x+y \end{pmatrix}$

We know that if ~~g~~ T is a linear transformation then $T(\vec{0}) = \vec{0}$

Choose $x = y = z = 0$

$$g\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 0+0 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \neq \vec{0}$$

Therefore this transformation is NOT linear.

(b) $h\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} xy \\ x+y \end{pmatrix}$

We know ~~tran~~ a transformation (or mapping) T is linear if

(i) $T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$ for all $\vec{u}$ & $\vec{v}$ in the domain of T.

(ii) $T(c\vec{u}) = cT(\vec{u})$ for all scalars c & $\vec{u}$ in the domain of T.

Here $h\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} xy \\ x+y \end{pmatrix}$ given.

1st element

Therefore $h(\vec{u}+\vec{v}) = h\begin{pmatrix} u_1+v_1 \\ u_2+v_2 \\ u_3+v_3 \end{pmatrix} \underline{\underline{\text{by defn.}}} \begin{pmatrix} (u_1+v_1)(u_2+v_2) \\ (u_1+v_1)+(u_2+v_2) \end{pmatrix}$

2nd element

3rd element.

1st element   2nd element

1st element   2nd element

$$= \begin{pmatrix} (u_1+v_1)(u_2+v_2) \\ (u_1+u_2)+(v_1+v_2) \end{pmatrix}$$

$$h(\vec{u}) = h\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} u_1u_2 \\ u_1+u_2 \end{pmatrix}$$

$$h(\vec{v}) = h\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} v_1v_2 \\ v_1+v_2 \end{pmatrix}$$

$$h(\vec{u}) + h(\vec{v}) = \begin{pmatrix} u_1u_2 + v_1v_2 \\ u_1+u_2+v_1+v_2 \end{pmatrix}$$

$$\Rightarrow h(\vec{u}+\vec{v}) \neq h(\vec{u}) + h(\vec{v})$$

Since $\begin{pmatrix} (u_1+v_1)(u_2+v_2) \\ (u_1+u_2)+(v_1+v_2) \end{pmatrix} \neq \begin{pmatrix} u_1u_2+v_1v_2 \\ u_1+u_2+v_1+v_2 \end{pmatrix} \Rightarrow h$ is <u>NOT</u> a linear transformation

8(c) $f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z-x \\ x+y \end{pmatrix}$

We will use same criteria for linear transform. as is shown in 8(b).

(b) Let $\vec{u}$ & $\vec{v}$ are in domain of f.

(i) Then $f(\vec{u}) = \begin{pmatrix} u_3-u_1 \\ u_1+u_2 \end{pmatrix}$ $\quad f(\vec{v}) = \begin{pmatrix} v_3-v_1 \\ v_1+v_2 \end{pmatrix}$

$$\Rightarrow f(\vec{u}+\vec{v}) = \begin{pmatrix} (u_3-u_1)+(v_3-v_1) \\ (u_1+u_2)+(v_1+v_2) \end{pmatrix}$$

$$\Rightarrow f\begin{pmatrix} \underbrace{u_1+v_1}_{\text{1st element}} \\ \underbrace{u_2+v_2}_{\text{2nd element}} \\ \underbrace{u_3+v_3}_{\text{3rd element}} \end{pmatrix} = \begin{pmatrix} u_3-u_1+v_3-v_1 \\ u_1+u_2+v_1+v_2 \end{pmatrix}$$

$$= \begin{pmatrix} (u_3+v_3)-(u_1+v_1) \\ (u_1+v_1)+(u_2+v_2) \end{pmatrix}$$

$$= \begin{pmatrix} \text{3rd element} - \text{1st element} \\ \text{1st element} + \text{2nd element} \end{pmatrix}$$

Therefore the 1st criterion is fulfilled.

(ii) Let c be any scalar.

$$f(c\vec{u}) = f\begin{pmatrix} cu_1 \\ cu_2 \\ cu_3 \end{pmatrix} = \begin{pmatrix} cu_3 - cu_1 \\ cu_1+cu_2 \end{pmatrix} = c\begin{pmatrix} u_3-u_1 \\ u_1+u_2 \end{pmatrix} = c f(\vec{u})$$

Note: $c f(\vec{u}) = c\begin{pmatrix} u_3-u_1 \\ u_1+u_2 \end{pmatrix}$

$\Rightarrow f(c\vec{u}) = c f(\vec{u})$ Since both criteria ~~Therefore~~ are satisfied $\Rightarrow f$ is a linear transform.