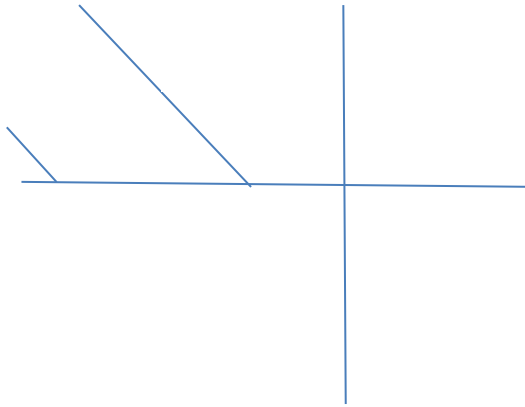


In exercise 23 and 24 mark each statement true or false. Justify each answer.

#23

- a. Another notation for the vector $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ is $-4 \ 3$. False since $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ is a 2×1 matrix and $-4 \ 3$ is a 1×2 matrix.
- b. The point in the plane corresponding to $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$ lie on a line through the origin. False



- c. An example of a linear combination of vector V_1 and V_2 is the vector $1/2V_1$. True
- d. The solution set of the linear system whose augmented matrix is $[a_1 \ a_2 \ a_3 \ b]$ is the same as the solution set of the equation $x_1a_1 + x_2a_2 + x_3a_3 = b$ True
- e. The set $\text{Span}\{u, v\}$ is always visualized as a plane through the origin. FALSE

25. Let $A = \begin{pmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 3 \end{pmatrix}$ and $b = \begin{pmatrix} 4 \\ 1 \\ -4 \end{pmatrix}$. Denote the columns of A by a_1, a_2, a_3 , and let $W = \text{Span}\{a_1, a_2, a_3\}$

a. Is b in $\{a_1, a_2, a_3\}$? How many vectors are in $\{a_1, a_2, a_3\}$?

No, b is not in a_1, a_2, a_3 . There are 3 vectors

b. Is b in W ? How many vectors are in W ?

c. Show that a_1 is in W

$$a_1 = 1 * a_1 + 0 * a_2 + 0 * a_3$$

27.

A. Mining Company has two mines. One day's operation at mine #1 produces ore that contain 30 metric tons of copper and 600 kilograms of silver, while one day's operation at mine #2 produces ore that contains 40 metric tons of copper and 380 kilograms of silver. Let $V_1 = \begin{pmatrix} 30 \\ 600 \end{pmatrix}$ and $V_2 = \begin{pmatrix} 40 \\ 380 \end{pmatrix}$. Then V_1 and V_2 represent the "output per day" of mine #1 and mine #2, respectively

a. What physical interpretation can be given to vector $5V_1$

It means 5 day output of v_1

b. Suppose the company operates mine #1 for X_1 day and mine #2 operate for X_2 days. Write a vector equation whose solution gives the number of day each mine should operate in order to produce 240 tons of copper and 2824 kilograms of silver. Do not solve the equation.

$$X_1V_1 + X_2V_2 = \begin{pmatrix} 240 \\ 2824 \end{pmatrix}$$

c. Solve equation in B

$$X_1 \frac{30}{600} + X_2 \frac{40}{380} = \frac{240}{2824}$$

$$X_1 30 + X_2 40 = 240$$

$$X_1 600 + X_2 380 = 2824$$

$$(X_1 30 + X_2 40 = 240) \times 20$$

$$X_1 600 + X_2 800 = 4800$$

$$\underline{\quad \quad \quad - - X_1 600 + X_2 380 = 2824 \quad \quad \quad}$$

$$X_2 420 = 1976$$

$$X_2 = 4.70$$

$$X_1 = 1.73$$

$$1. \begin{pmatrix} -4 & 2 & 3 \\ 1 & 6 & -2 \\ 0 & 1 & 7 \end{pmatrix} = \text{Undefined}$$

$$3 \times 2 \neq 3 \times 1$$

5. Use the definition of matrix of Ax to write a the matrix equation as a vector equation, or vice versa.

$$\begin{pmatrix} 1 & 2 & -3 & 1 \\ -2 & -3 & 1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$$

$$\begin{aligned} & (1)2 + (-1)2 + (1)(-3) + (-1)(1) \\ & -2 \cdot 2 + (-3) \cdot (-1) + 1 \cdot 1 + (-1)(-1) \end{aligned}$$

$$\begin{aligned} & 2 + (-2) + (-3) + (-1) = -4 \\ & -4 + 3 + 1 + 1 = 1 \end{aligned}$$

17. How many rows of A contain a pivot position? Does the equation $Ax = b$ have a solution for each b in R^4 ?

$$A = \begin{pmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 2 & 6 & 7 \\ 2 & 9 & 5 & -7 \end{pmatrix}$$

$$1(R_1) + R_2 \rightarrow R_2 \quad \begin{pmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$-2R_1 + R_4 \rightarrow R_4 \quad \begin{pmatrix} 1 & 3 & 0 & 3 \\ 0 & 2 & -1 & 4 \\ 0 & -4 & 2 & -8 \\ 0 & -6 & 3 & -7 \end{pmatrix}$$

$$\frac{1}{2}R_2 \rightarrow R_2 \quad \begin{pmatrix} 1 & 3 & 0 & 3 \\ 0 & 1 & -\frac{1}{2} & 2 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{pmatrix}$$

$$-3R_2 + R_1 \rightarrow R_1 \quad \begin{pmatrix} 1 & 0 & -3/2 & 3 \\ 0 & 1 & -1/2 & 2 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{pmatrix}$$

$$4R_2 + R_3 \rightarrow R_3 \quad \begin{pmatrix} 1 & 0 & -3/2 & 3 \\ 0 & 1 & -1/2 & 2 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & 3 & -1 \end{pmatrix}$$

$$6R_2 + R_4 \rightarrow R_4 \quad \begin{array}{cccc} 1 & 0 & -3/2 & 3 \\ 0 & 1 & -1/2 & 2 \\ 0 & 0 & 0 & 0 \\ 2 & 6 & 0 & 11 \end{array}$$

19. Can each vector in \mathbb{R}^4 be written as a linear combination of the columns of the matrix A above? Do the columns of A span \mathbb{R}^4 ?