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Exercises from 1-21 odd

1a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ Reduced Equelon Form

1b) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ Reduced Equelon Form

1c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ Not in Equelon Form

1d) $\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$ In Equelon Form

3) $\begin{bmatrix} 1 & 2 & 4 & 8 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 2 \end{bmatrix}$

Multiply row 2 by $-1/2$

$$\begin{array}{cccc} 1 & 2 & 4 & 8 \\ -1 & -2 & -3 & -4 \\ \hline 0 & 0 & 1 & 4 \end{array}$$

Multiply row 3 by $-1/3$

$$\begin{array}{cccc} 1 & 2 & 4 & 8 \\ -1 & -2 & -3 & -4 \\ \hline 0 & 0 & 1 & 4 \end{array}$$

We get: $\begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ but now add -4 times row 2 to row 1 and add -1 times row 2 to row 3

$$\begin{array}{cccc} 1 & 2 & 4 & 8 \\ 0 & 0 & -4 & -16 \\ \hline 1 & 2 & 0 & -8 \end{array}$$

$$\begin{array}{cccc} 0 & 0 & 1 & 4 \\ 0 & 0 & -1 & -4 \\ \hline 0 & 0 & 0 & 0 \end{array}$$

Reduced Equelon Form: $\begin{bmatrix} 1 & 2 & 0 & -8 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

5) Possible echelon forms on a non-zero 2X2 matrix:

$\begin{bmatrix} \blacksquare & * \\ 0 & \blacksquare \end{bmatrix}$ or $\begin{bmatrix} \blacksquare & * \\ 0 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & \blacksquare \\ 0 & 0 \end{bmatrix}$

7) $\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix}$

$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 0 & 0 & \frac{5}{3} & 5 \end{bmatrix} \leftarrow row\ 1 + (-\frac{1}{3} \cdot\ row2)$ , therefore:

$$x_1 = -3x_2 - 12 + 7 = -5 - 3x_2$$

$$x_3 = \frac{3}{5} \cdot 5 = 3$$

$$x_2\ is\ free$$

9) $\begin{bmatrix} 0 & 1 & -2 & 3 \\ 1 & -3 & 4 & -6 \end{bmatrix}$ $Switch\ row1\ and\ row\ 2 \rightarrow \begin{bmatrix} 1 & -3 & 4 & -6 \\ 0 & 1 & -2 & 3 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & -2 & 3 \end{bmatrix} \leftarrow\ row1 + (row2\ \cdot\ 3)$, therefore:

$$x_1 - 2x_3 = 3;\ x_1 = 2x_3 + 3$$

$$x_2 - 2x_3 = 3;\ x_2 = 2x_3 + 3$$

$$x_3\ is\ free$$

11) $\begin{bmatrix} 3 & -2 & 4 & 0 \\ 9 & -6 & 12 & 0 \\ 6 & -4 & 8 & 0 \end{bmatrix}$

$\begin{bmatrix} 3 & -2 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \leftarrow row2 + (-3 \cdot row1) \\ \leftarrow row3 + (-2 \cdot row1) \end{matrix}$

$\begin{bmatrix} 1 & -\frac{2}{3} & \frac{4}{3} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \leftarrow scale\ row1\ by\ \frac{1}{3}$

$$x_1 - \frac{2}{3}x_2 + \frac{4}{3}x_3 = 0;\ x_1 = \frac{2}{3}x_2 - \frac{4}{3}x_3$$

$$x_2\ is\ free$$

$$x_3\ is\ free$$

13) $\begin{bmatrix} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & -12 & 1 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \leftarrow Row1 + (3 \cdot row2)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -3 & 5 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \leftarrow Row1 + row3$$

$$x_1 - 3x_5 = 5;\ x_1 = 5 + 3x_5$$

$$x_2 - 4x_5 = 1;\ x_2 = 1 + 4x_5$$

$$x_3\ is\ free$$

$$x_4 + 9x_5 = 4;\ x_4 = 4 - 9x_5$$

$$x_5\ is\ free$$

15) Both matrices are consistent. For a consistent matrix with a unique solution, the matrix should not have free variables.

$$\text{a.}\ \begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{b.}\ \begin{bmatrix} 0 & \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * & * \\ 0 & 0 & 0 & \blacksquare & 0 \end{bmatrix}$$

In matrix [a] we can observe that $x_3$ is a free variable. Matrix [b] has $x_1$ as free variable.

17) $\begin{bmatrix} 1 & -1 & 4 \\ -2 & 3 & h \end{bmatrix}$

$$\begin{array}{ccc} 1 & -1 & 4 \\ -1 & \frac{3}{2} & \frac{h}{2} \\ \hline 0 & \frac{1}{2} & 4\frac{h}{2} \end{array}$$

$$\begin{bmatrix} 1 & -1 & 4 \\ 0 & \frac{1}{2} & 4\frac{h}{2} \end{bmatrix} \leftarrow row1 + (2 \cdot row2)$$

$$\begin{bmatrix} 1 & -1 & 4 \\ 0 & 1 & 4 + h \end{bmatrix} \leftarrow Scale\ by\ 2$$

$$\begin{bmatrix} 1 & 0 & 8 + h \\ 0 & 1 & 4 + h \end{bmatrix}$$

There is not a constant value for h. h may vary in value, reason why we say that h can be any value.

19) $\begin{bmatrix} 1 & h & 2 \\ 4 & 8 & k \end{bmatrix}$

$$\begin{array}{ccc} 1 & h & 2 \\ -1 & -2 & -\frac{k}{4} \\ \hline 0 & h-2 & 2-\frac{k}{4} \end{array}$$

$$\begin{bmatrix} 1 & h & 2 \\ 0 & h-2 & 2-\frac{k}{4} \end{bmatrix}$$

We can conclude that when h = 2 and k ≠ 8 the matrix is inconsisted

When h ≠ 2 the solution is unique

When h = 2 and k = 8 there are many solutions

21a) True
21b) False
21c) True
21d) *not sure…
21e) True