Victor Ng – 9/11/2012

Determine if the systems in Exercises 15 and 16 are consistent. Do not completely solve the systems.

15.
$$\begin{aligned} x_1 - 6x_2 \qquad\qquad &= 5 \\ x_2 - 4x_3 + x_4 &= 0 \\ -x_1 + 6x_2 + x_3 + 5x_4 &= 3 \\ -x_2 + 5x_3 + 4x_4 &= 0 \end{aligned}$$

<u>Augmented Matrix</u>

$$\begin{bmatrix} 1 & -6 & 0 & 0 & | & 5 \\ 0 & 1 & -4 & 1 & | & 0 \\ -1 & 6 & 1 & 5 & | & 3 \\ 0 & -1 & 5 & 4 & | & 0 \end{bmatrix} R_3 - (-1 * R_1) = R_3 \begin{bmatrix} 1 & -6 & 0 & 0 & | & 5 \\ 0 & 1 & -4 & 1 & | & 0 \\ 0 & 0 & 1 & 5 & | & 8 \\ 0 & -1 & 5 & 4 & | & 0 \end{bmatrix}$$

17. Do the three lines $2x_1 + 3x_2 = -1$, $6x_1 + 5x_2 = 0$, and $2x_1 - 5x_2 = 7$ have a common point of intersection? Explain.

$2x_1+3x_2 = -1$

$6x_1+5x_2 = 0$

$2x_1-5x_2 = 7$

In Exercises 19–22, determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system.

19. $\begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix}$ **21.** $\begin{bmatrix} 1 & 4 & -2 \\ 3 & h & -6 \end{bmatrix}$

23. a. Every elementary row operation is reversible.

b. A 5×6 matrix has six rows.

c. The solution set of a linear system involving variables $x_1, \ldots, x_n$ is a list of numbers $(s_1, \ldots, s_n)$ that makes each equation in the system a true statement when the values $s_1, \ldots, s_n$ are substituted for $x_1, \ldots, x_n$, respectively.

d. Two fundamental questions about a linear system involve existence and uniqueness.

A) True.

B) False, A 5x6 matrix has 5 rows and 6 columns.

C) False, describes only one element but not entire solution set.

D) True.

25. Find an equation involving g, h, and k that makes this augmented matrix correspond to a consistent system:

$$\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{bmatrix}$$

K-2G+H = 0

For values of Row 1 : (-2) – (-2*1) + 0 = 0

-2 + 2 = 0

0 = 0 Therefore system is consistent.

27. Suppose a, b, c, and d are constants such that a is not zero and the system below is consistent for all possible values of f and g. What can you say about the numbers a, b, c, and d? Justify your answer.

$$\begin{aligned} ax_1 + bx_2 &= f \\ cx_1 + dx_2 &= g \end{aligned}$$

In Exercises 29–32, find the elementary row operation that transforms the first matrix into the second, and then find the reverse row operation that transforms the second matrix into the first.

29. $\begin{bmatrix} 0 & -2 & 5 \\ 1 & 3 & -5 \\ 3 & -1 & 6 \end{bmatrix}, \begin{bmatrix} 3 & -1 & 6 \\ 1 & 3 & -5 \\ 0 & -2 & 5 \end{bmatrix}$

31. $\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 4 & -1 & 3 & -6 \end{bmatrix}, \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 5 & -2 & 8 \\ 0 & 7 & -1 & -6 \end{bmatrix}$

29) Swap R_3 with R_1

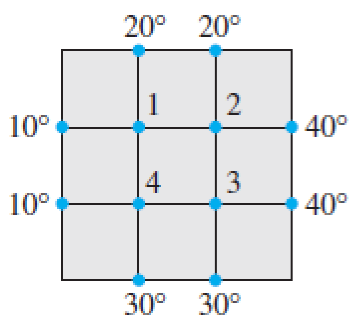
$\begin{bmatrix} 0 & -2 & 5 \\ 1 & 3 & -5 \\ 3 & -1 & 6 \end{bmatrix}$  $\begin{bmatrix} 3 & -1 & 6 \\ 1 & 3 & -5 \\ 0 & -2 & 5 \end{bmatrix}$

31) Replace R_3 with (R_3 + (-4*R_1)).

$$\begin{bmatrix} \mathbf{1} & \mathbf{-2} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{5} & \mathbf{-2} & \mathbf{8} \\ \mathbf{4+1(-4)} & \mathbf{3+1(-4)} & \mathbf{3+1(-4)} & \mathbf{-6+0(-4)} \end{bmatrix}$$

An important concern in the study of heat transfer is to determine the steady-state temperature distribution of a thin plate when the temperature around the boundary is known. Assume the plate shown in the figure represents a cross section of a metal beam, with negligible heat flow in the direction perpendicular to the plate. Let $T_1, \ldots, T_4$ denote the temperatures at the four interior nodes of the mesh in the figure. The temperature at a node is approximately equal to the average of the four nearest nodes—to the left, above, to the right, and below.[3] For instance,

$$T_1 = (10 + 20 + T_2 + T_4)/4, \quad \text{or} \quad 4T_1 - T_2 - T_4 = 30$$



33. Write a system of four equations whose solution gives estimates for the temperatures $T_1, \ldots, T_4$.

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In Exercises 9 and 10, write a vector equation that is equivalent to the given system of equations.

9.
$$\begin{aligned} x_2 + 5x_3 &= 0 \\ 4x_1 + 6x_2 - x_3 &= 0 \\ -x_1 + 3x_2 - 8x_3 &= 0 \end{aligned}$$

$$\mathbf{X1}\begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} + \mathbf{X2}\begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix} + \mathbf{X3} = \begin{bmatrix} 5 \\ -1 \\ -8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$